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TRAINING AND PLACEMENT CELL



APTITUDE
AND
LOGICAL REASONING



APTITUDE & LOGICAL REASONING

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MATHEMATICS
is not about
numbers, equations,
computations, or
algorithms:
it is about
UNDERSTANDING

SECTION - I

**QUANTITATIVE
APTITUDE**

1. NUMBER SYSTEM

A system in which different types of numbers, their relationship and rule govern in them is called as number system.

NUMBERS

In the Hindu-Arabic system, we use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These symbols are called as digits. To write a number we put digits from right to left at the places designated as units, tens, hundreds, thousands, ten thousands, lakhs, ten lakhs, crore, ten crores.

FACE VALUE

Face value of a digit in a numeral is value of the digit itself irrespective of its place in the numeral. For example in 321, face value of 1 is 1, face value of 2 is 2 and face value of 3 is 3.

DRILL

- Find the face value of 6 in 652410.

a) 6×10^5	b) 6×10^4	c) 6000	d) 6
--------------------	--------------------	---------	------
- Find the difference of the face values of 7 and 2 in 210978.

a) 4	b) 3	c) 6	d) 5
------	------	------	------
- Find the sum of the face values of 9 and 6 in 907364.

a) 15	b) 20	c) 9	d) 18
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PLACE VALUE

In a numeral, the place value of a digit changes according to the change of its place. Place value of a digit in a numeral is value of the digit multiplied by 10^n where n starts from 0.

For example in 654:

- Place value of 4 = $4 \times 100 = 4 \times 1 = 4$
- Place value of 5 = $5 \times 101 = 5 \times 10 = 50$
- Place value of 6 = $6 \times 102 = 6 \times 100 = 600$

0th position digit is called as unit digit.

DRILL

- Find the place value of 4 in 46127.

a) 4	b) 400	c) 40000	d) 4000
------	--------	----------	---------
- What is the place value of 6 in 65489203?

a) 6×10^5	b) 6×10^4	c) 6×10^7	d) 6×10^8
--------------------	--------------------	--------------------	--------------------
- The sum of place values of 2 in 2424 is

a) 4	b) 220	c) 2002	d) 2020
------	--------	---------	---------
- Find the sum of place value of 6 and face value of 9 in 927653.

a) 608	b) 508	c) 609	d) 507
--------	--------	--------	--------

TYPES OF NUMBERS

1. Natural Numbers

Natural numbers are counting numbers. They are denoted by N.

For example,

$$N = \{1, 2, 3, \dots\}.$$

- All natural numbers are positive.
- Zero is not a natural number. Therefore, 1 is the smallest natural number.

2. Whole Numbers

All natural numbers and zero form the set of whole numbers.

Whole numbers are denoted by W.

For example,

$$W = \{0, 1, 2, 3, \dots\}$$

- Zero is the smallest whole number.
- Whole numbers are also called as non-negative integers.

3. Integers

Whole numbers and negative numbers form the set of integers. They are denoted by I.

For example,

$$I = \{-\infty, \dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots, \infty\}$$

Integers are of two types,

- Positive Integers Natural numbers are called as positive integers. They are denoted by I+.

For example,

$$I+ = \{1, 2, 3, 4, \dots\}$$

- Negative Integers Negative of natural numbers are called as negative integers. They are denoted by I–

For example,

$$I- = \{-1, -2, -3, -4, \dots\}$$

- '0' is neither positive nor negative integer.

4. Even Numbers

A counting number which is divisible by 2, is called an even number. The unit's place of every even number will be 0, 2, 4, 6 or 8.

For example 2, 4, 6, 8, 10, 12, 14, ... etc.

5. Odd Numbers

A counting number which is not divisible by 2, is known as odd number. The unit's place of every odd number will be 1, 3, 5, 7 or 9.

For example 1, 3, 5, 7, 9, 11, 13, ... etc.

6. Prime Numbers

A counting number is called a prime number when it is exactly divisible by 1 and itself.
For example 2, 3, 5, 7, 11, 13,... etc.

- 2 is the only even number which is prime.
- A prime number is always greater than 1.
- 1 is not a prime number. Therefore, the lowest odd prime number is 3.

To find a prime number

Example 1: 191 is prime number or not?

Step 1: $14 > \sqrt{191}$

Step 2: Prime numbers less than 14 are 2, 3, 5, 7, 11 and 13.

Step 3: 191 is not divisible by any above prime number.

Result: 191 is a prime number.

Example 2: 187 is prime number or not?

Step 1: $14 > \sqrt{187}$

Step 2: Prime numbers less than 14 are 2, 3, 5, 7, 11 and 13.

Step 3: 187 is divisible by 11.

Result: 187 is not a prime number.

7. Composite Numbers

Composite numbers are non-prime natural numbers. They must have atleast one factor apart from 1 and itself.

For example 4, 6, 8, 9, 10, 12,... etc.

- Composite numbers can be both odd and even.
- 1 is neither a prime number nor composite number.

8. Co-primes

Two natural numbers are said to be co-primes, if their HCF is 1.

For example: (13, 11), (9, 8), (5, 12).

- Co-prime numbers may or may not be prime.

9. Rational Numbers

A number that can be expressed as p/q is called a rational number, where p and q are integers and $q \neq 0$.

For example: $\frac{3}{5}$, $\frac{7}{15}$, $\frac{17}{13}$, $\frac{20}{23}$

10. Irrational Numbers

The numbers that cannot be expressed in the form of p/q are called irrational numbers, where p and q are integers and $q \neq 0$.

For example $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$, $\sqrt{11}$

- π is an irrational number as $22/7$ is not the actual value of π but it is its nearest value.
- Non-periodic infinite decimal fractions are called as irrational number.

DRILL

1. The pair of numbers which are relatively prime to each other is
 a) (68, 85) b) (65, 91) c) (92, 85) d) (102, 153)
2. Find the sum of 1st and 2nd prime numbers.
 a) 5 b) 3 c) 7 d) 2
3. Find the product of 1st natural number and 1st prime number.
 a) 4 b) 3 c) 2 d) 5
4. The product of 1st natural, 1st whole and the 1st prime numbers is equal to
 a) 5 b) 0 c) 9 d) 7
5. The product of any number and the 1st whole number is equal to
 a) 0 b) 2 c) 1 d) -1
6. A rational number is expressed as _____ where, p and q are integers and $q \neq 0$.
 a) pq b) $p + q$ c) $p - q$ d) $\frac{p}{q}$
7. How many rational numbers are there between 1 and 1000?
 a) 998 b) 999 c) 1000 d) Infinite
8. $\frac{2}{3}$ is a rational number whereas $\sqrt{2} / \sqrt{3}$ is
 a) also a rational number b) an irrational number
 c) not a number d) a natural periodic number
9. The product of four consecutive natural numbers plus one is
 a) a non-square b) always sum of two square numbers
 c) a square d) None of the above
10. Which of the following is a prime number?
 a) 119 b) 53 c) 87 d) 91

DIVISIBILITY RULE**1. Divisibility by 2**

When the last digit of a number is either 0 or even, then the number is divisible by 2.

Example 1: 334 is divisible by 2 or not?

Step 1: Unit digit is 4.

Result: 334 is divisible by 2.

Example 2: 563 is divisible by 2 or not?

Step 1: Unit digit is 3.

So, the given number 563 is not divisible by 2.

2. Divisibility by 3

When sum of the digits of a number is divisible by 3, then the number is divisible by 3.

Example 1: 5742 is divisible by 3 or not?

Step 1: Sum of its digits is $5 + 7 + 4 + 2 = 18$

18 is divisible by 3.

So, the given number 5742 is divisible by 3.

Example 2: 4573 is divisible by 3 or not?

Step 1: Sum of its digits is $4 + 5 + 7 + 3 = 19$

19 is not divisible by 3.

So, the given number 4573 is not divisible by 3.

3. Divisibility by 4

If last two-digits of a number is divisible by 4, then that number is divisible by 4. Apart from this, the number having two or more zeroes at the end is also divisible by 4.

Example 1: 7248 is divisible by 4 or not?

Step 1: Last two digits of the given number is 48

48 is divisible by 4.

So, the given number 7248 is divisible by 4.

Example 2: 6531 is divisible by 4 or not?

Step 1: Last two digits of the given number is 31

31 is not divisible by 4.

So, the given number 6531 is not divisible by 4.

4. Divisibility by 5:

A number is divisible by 5 if its unit digit is 0 or 5.

Example 1: 775 is divisible by 5 or not?

Step 1: Unit digit is 5.

So, the given number 775 is divisible by 5.

Example 2: 578 is divisible by 5 or not?

Step 1: Unit digit is 8.

So, the given number 578 is not divisible by 5.

5. Divisibility by 6

A number is divisible by 6 if the number is divisible by both 2 and 3.

Example 1: 216 is divisible by 6 or not?

Step 1: Unit digit is 6. Number is divisible by 2.

Step 2: Sum of its digits is $2 + 1 + 6 = 9$

9 is divisible by 3.

So, the given number 216 is divisible by 6.

Example 2: 452 is divisible by 6 or not?

Step 1: Unit digit is 2.

So, the given number 452 is is divisible by 2.

Step 2: Sum of its digits is $4 + 5 + 2 = 11$

11 is not divisible by 3.

So, the given number 452 is not divisible by 6.

6. Divisibility by 7

A number is divisible by 7 when the difference between twice the digit at ones place and the number formed by other digits is either zero or a multiple of 7.

Example 1: 658 is divisible by 7 or not?

Step 1: $65 - (2 \times 8) = 65 - 16 = 49$.

49 is divisible by 7

So, the given number 658 is divisible by 7.

Example 2: 423 is divisible by 7 or not?

Step 1: $42 - (2 \times 3) = 42 - 6 = 36$.

36 is not divisible by 7

So, the given number 423 is not divisible by 7.

7. Divisibility by 8

If the last three-digits of a number is divisible by 8, then that particular number is divisible by 8. Apart from this, the number having three or more zeroes at the end is also divisible by 8.

Example 1: 14568 is divisible by 8 or not?

Step 1: Last three digits of the given number is 568

568 is divisible by 8.

So, the given number 14568 is divisible by 8.

Example 2: 6,40,000 is divisible by 8 or not?

Step 1: Last three digits of the given number is 000

So 6,40,000 is divisible by 8.

8. Divisibility by 9

If sum of the digits of a number is divisible by 9, then the number is divisible by 9.

Example 1: 4473 is divisible by 9 or not?

Step 1: Sum of its digits is $4 + 4 + 7 + 3 = 18$

18 is divisible by 9.

So, the given number 4473 is divisible by 9.

Example 2: 4579 is divisible by 9 or not?

Step 1: Sum of its digits is $4 + 5 + 7 + 9 = 25$

25 is not divisible by 9.

So, the given number 4579 is not divisible by 9.

9. Divisibility by 10

A number is divisible by 10 if its unit digit is 0.

Example 1: 25670 is divisible by 10 or not?

Step 1: Unit digit is 0.

So, 25670 is divisible by 10.

Example 2: 1575 is divisible by 10 or not?

Step 1: Unit digit is 5.

So, 1575 is not divisible by 10.

10. Divisibility by 11

A number is divisible by 11 if difference between sum of digits at odd places and sum of digits at even places is either 0 or multiples of 11.

Example 1: 93819 is divisible by 11 or not?

Step 1: Difference between sum of digits at odd places and sum of digits at even places

$$= (9+8+9) - (3+1) = 22$$

22 is divisible by 11.

So, the given number 93819 is divisible by 11.

Example 2: 64075 is divisible by 11 or not?

Step 1: Difference between sum of digits at odd places and sum of digits at even places

$$= (6+0+5) - (4+7) = 0.$$

So, the given number 64075 is divisible by 11.

11. Divisibility by 12

A number is divisible by 12 if the number is divisible by both 4 and 3.

Example: 2244 is divisible by 12 or not?

Step 1: Last two digits of the given number is 44

44 is divisible by 4.

Step 2: Sum of its digits is $2 + 2 + 4 + 4 = 12$

12 is divisible by 3.

So, the given number 2244 is divisible by 12.

Division Algorithm

When a number is divided by another number then

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Reminder}$$

DRILL

- Find the dividend from the expression $25\frac{3}{5}$
 - 128
 - 143
 - 122
 - 155
- Find the dividend when divisor is 13, quotient is 30 and remainder is 12.
 - 402
 - 543
 - 436
 - 455

3. In a question on division with zero remainder, a candidate took 12 as divisor instead of 21. The quotient obtained by him was 35. Find the correct quotient.
a) 10 b) 12 c) 20 d) 15
4. In a division sum, the divisor is ten times the quotient and five times the remainder. If the remainder is 46, then find the dividend.
a) 5388 b) 5343 c) 5336 d) 5391
5. The number 58129745812974 is divisible by
a) 11 b) 9 c) 4 d) None of these
6. What least number must be added to 1057 to get a number exactly divisible by 23?
a) 1 b) 3 c) 2 d) 4
7. What least number should be added to 12548467 to make it divisible by 8?
a) 3 b) 4 c) 6 d) 5
8. What number should be subtracted to 231235 to make it exactly divisible by 33?
a) 1 b) 2 c) 3 d) 4
9. Which of the following numbers always divides the difference between the squares of two consecutive odd integers?
a) 7 b) 3 c) 8 d) 6
10. A number divided by 56 gives 29 as remainder. Find the remainder, if the same number is divided by 8.
a) 4 b) 5 c) 6 d) 7
11. On dividing a certain number by 357, the remainder is 39. On dividing the same number by 17, what will be the remainder?
a) 5 b) 3 c) 7 d) 6
12. A number when divided by 5 leaves 3 as remainder. What will be the remainder when the square of this number is divided by 5?
a) 3 b) 4 c) 5 d) 0
13. A number when divided by a divisor leaves a remainder of 24. When twice the original number is divided by the same divisor, the remainder is 11. What is the value of divisor?
a) 13 b) 59 c) 35 d) 37
14. How many numbers between -11 and 11 are multiples of 2 or 3?
a) 11 b) 14 c) 15 d) None of these

2. UNIT DIGIT

Unit digit of a number is the digit in the one's place of the number. The concept of unit digit can be learned by figuring out the unit digits of all the single digit numbers from 0 to 9.

Example:

What will be unit digit in $658 \times 539 \times 436 \times 312$?

- (a) 8 (b) 9 (c) 4 (d) 6

Solution:

Multiply unit digits of each number.

- Unit digit in $658 \times 539 \times 436 \times 312$
 = Unit digit in $8 \times 9 \times 6 \times 2$.
 = Unit digit in 864.
 = 4.

DRILL

- Find the unit digit in the product ($784 \times 618 \times 917 \times 463$)
 a) 2 b) 3 c) 4 d) 5
- Find the unit digit of the sum of first 150 whole numbers.
 a) 9 b) 5 c) 0 d) 1
- Find the unit digit in $1! + 2! + 3! + 4! + \dots + 50!$
 a) 1 b) 3 c) 5 d) 8

CYCLICITY OF NUMBERS

The cyclicity of any number is mainly focused on its unit digit. Every unit digit has its own repetitive pattern when raised to any power.

These numbers can be broadly classified into three categories listed as follows:

1. For digits 0, 1, 5, and 6

When each of these digits is raised to any power, the unit digit of the final answer is the number itself.

Examples:

- $0^4 = 0$: Unit digit is 0, the number itself.
- $1^6 = 1$: Unit digit is 1, the number itself.
- $5^2 = 25$: Unit digit is 5, the number itself.
- $6^3 = 216$: Unit digit is 6, the number itself.

2. For digits 4 and 9

Both of these two digits 4 and 9 have a cyclicity of two different digits as their unit digit.

Examples:

- $4^2 = 16$: Unit digit is 6. $4^{\text{even}} =$ Unit digit is 6.
- $4^3 = 64$: Unit digit is 4. $4^{\text{odd}} =$ Unit digit is 4.
- $9^2 = 81$: Unit digit is 1. $9^{\text{even}} =$ Unit digit is 1.
- $9^3 = 729$: Unit digit is 9. $9^{\text{odd}} =$ Unit digit is 9.

3. For digits 2, 3, 7, and 8

These numbers have a cyclicity of four different digits as their unit digit.

Examples:

- $2^1 = 2$: Unit digit is 2.
- $2^2 = 4$: Unit digit is 4.
- $2^3 = 8$: Unit digit is 8.
- $2^4 = 16$: Unit digit is 6.
- $2^5 = 32$: Unit digit is 2.
- $2^6 = 64$: Unit digit is 4.

It can be observed that the unit digit 2, 4, 8, 6 repeats themselves after a period of four numbers. Similarly,

- The cyclicity of 3 has 4 different numbers: 3, 9, 7, 1.
- The cyclicity of 7 has 4 different numbers: 7, 9, 3, 1.
- The cyclicity of 8 has 4 different numbers: 8, 4, 2, 4.

The concepts discussed above can be summarized as:

Number	Cyclicity	Power Cycle
0	1	0
1	1	1
2	4	2, 4, 8, 6
3	4	3, 9, 7, 1
4	2	4, 6
5	1	5
6	1	6
7	4	7, 9, 3, 1
8	4	8, 4, 2, 6
9	2	9, 1

Example 1:

What will be unit digit in $(3157)^{754}$?

- a) 8 b) 9 c) 7 d) 6

Solution:

Unit digit of $(3157)^{754} = \text{unit digit of } (7)^{754}$

- The cyclicity of 7 has 4 different numbers: 7, 9, 3, 1.

$$754/4 = 188\frac{2}{4}$$

Remainder = 2

For remainder 2, unit digit is 9

∴ Unit digit in $(3157)^{754}$ is 9.

Example 2:

What will be unit digit in $3^{57} \times 6^{41} \times 7^{63}$?

- (a) 8 b) 9 c) 4 d) 6

Solution:

Unit digit of 3^{57}

- The cyclicity of 3 has 4 different numbers: 3, 9, 7, 1

$$57/4 = 14\frac{1}{4}$$

Remainder = 1

For remainder 1 unit digit is 3

∴ Unit digit in 3^{57} is 3.

Unit digit of 6^{41}

- The cyclicity of 6 has 6 only

∴ Unit digit in 6^{41} is 6.

Unit digit of 7^{63}

- The cyclicity of 7 has 4 different numbers: 7, 9, 3, 1.

$$63/4 = 15\frac{3}{4}$$

Remainder = 3

For remainder 3 unit digit is 3

∴ Unit digit in 7^{63} is 3.

So, Unit digit of $3^{57} \times 6^{41} \times 7^{63} = \text{Unit digit of } (3 \times 6 \times 3) = 4$

DRILL

- Find the last digit of 4^{55} .
a) 1 b) 2 c) 3 d) 4
- Find the last digit of 1547^{34} .
a) 5 b) 9 c) 7 d) 8
- Find the unit digit of $(432)^{412} \times (499)^{431}$
a) 8 b) 2 c) 4 d) 6
- If $X = (164)^{169} + (333)^{337} - (727)^{726}$, then find the unit digit of X.
a) 5 b) 9 c) 7 d) 8
- Find the unit digit of $[(25^{43} \times 56^{42}) + 456^{25} + 23^{42} + 76^{23}]$.
a) 1 b) 2 c) 3 d) 4
- Find the digit in the unit's place of the number represented by $(7^{95} - 3^{58})$.
a) 6 b) 7 c) 0 d) 4
- Find the unit digit of $6^1 + 6^3 + 6^5 + 6^7 + \dots + 6^{49}$
a) 0 b) 6 c) 1 d) 5
- What is the $1^{545} + 2^{545} + 3^{545} + 4^{545} + 5^{545} + 6^{545} + 7^{545} + 8^{545} + 9^{545}$?
a) 0 b) 6 c) 1 d) 5
- What is the unit digit in the product $(3^{65} \times 6^{59} \times 7^{71})$?
a) 1 b) 2 c) 4 d) 6
- Find the remainder when 3^{75} is divided by 5.
a) 1 b) 2 c) 3 d) 4
- Find the remainder when 7^{203} is divided by 4.
a) 1 b) 2 c) 3 d) 4
- Find the rightmost non-zero integer of the expression $1430^{343} + 1470^{367}$
a) 4 b) 7 c) 5 d) 3

3. SQUARE AND CUBE

SQUARE

Multiplication of any two same numbers is called as square. To square a number, just multiply it by itself.

Square values of 1 to 60

$1^2 = 1$	$21^2 = 441$	$41^2 = 1681$
$2^2 = 4$	$22^2 = 484$	$42^2 = 1764$
$3^2 = 9$	$23^2 = 529$	$43^2 = 1849$
$4^2 = 16$	$24^2 = 576$	$44^2 = 1936$
$5^2 = 25$	$25^2 = 625$	$45^2 = 2025$
$6^2 = 36$	$26^2 = 676$	$46^2 = 2116$
$7^2 = 49$	$27^2 = 729$	$47^2 = 2209$
$8^2 = 64$	$28^2 = 784$	$48^2 = 2304$
$9^2 = 81$	$29^2 = 841$	$49^2 = 2401$
$10^2 = 100$	$30^2 = 900$	$50^2 = 2500$
$11^2 = 121$	$31^2 = 961$	$51^2 = 2601$
$12^2 = 144$	$32^2 = 1024$	$52^2 = 2704$
$13^2 = 169$	$33^2 = 1089$	$53^2 = 2809$
$14^2 = 196$	$34^2 = 1156$	$54^2 = 2916$
$15^2 = 225$	$35^2 = 1225$	$55^2 = 3025$
$16^2 = 256$	$36^2 = 1296$	$56^2 = 3136$
$17^2 = 289$	$37^2 = 1369$	$57^2 = 3249$
$18^2 = 324$	$38^2 = 1444$	$58^2 = 3364$
$19^2 = 361$	$39^2 = 1521$	$59^2 = 3481$
$20^2 = 400$	$40^2 = 1600$	$60^2 = 3600$

Properties of Square

- Square of an even number is always an even number.
- Square of an odd number is always an odd number.
- It cannot be a negative number.
- If the unit digit of a number is 1, 5, 6 or 0 then the unit digit of their square value is same.
- It cannot end with 2, 3, 7 or 8.
- It cannot have odd number of zeros at its end.
- When we squaring a decimal number, the decimal place in the square value is twice that of the actual decimal places.

Squaring a number ending with 5

Step 1: Add one to the given number excluding the unit digit.

Step 2: Multiplying the given number excluding the unit digit with the step1 result.

Step 3: Square the unit digit i.e., $5^2 = 25$.

Step 4: Combine the results of step 2 followed by step 3 to obtain the required square value.

Example 1:

Find the value of 95^2

Step 1: $(9 + 1) = 10$

Step 2: $9 \times 10 = 90$

Step 3: $5^2 = 25$

Step 4: 9025

Example 2:

Find the value of 115^2

Step 1: $(11 + 1) = 12$

Step 2: $11 \times 12 = 132$

Step 3: $5^2 = 25$

Step 4: 13225

Squaring of a number**Example 1:**

Find the value of 76^2

Step 1: Write the given number excluding the unit digit and then square it.

➤ $7^2 = 49$

Step 2: Square the unit digit.

➤ $6^2 = 36$

Step 3: Multiplying the given number excluding the unit digit with the unit digit

➤ $7 \times 6 = 42$

Step 4: Multiply result of step 3 by 2.

➤ $42 \times 2 = 84$

Multiply it by 10,

➤ $84 \times 10 = 840$

Step 5:

➤ $4936 + 840 = 5776$

Example 2:

Find the value of 132^2

Step 1: Write the given number excluding the unit digit and then square it.

➤ $13^2 = 169$

Step 2: Square the unit digit.

➤ $2^2 = 04$

Step 3: Multiplying the given number excluding the unit digit with the unit digit

➤ $13 \times 2 = 26$

Step 4: Multiply result of step 3 by 2.

➤ $26 \times 2 = 52$

Multiply it by 10,

➤ $52 \times 10 = 520$

Step 5:

➤ $16904 + 520 = 17424$

SQUARE ROOT

The square root of a number is a value that gives the original number, when it is multiplied by itself.

Example 1:

Find the square root of 729.

Solution:

Step 1:

Split the last two digits from the given number and take the remaining digits.

$$7 _ 29$$

Step 2:

Find the nearest perfect square less than the step 1 result.

Take the square root for that number. That is the first digit of the required value.

i.e., Perfect square less than 7 is $4 = 2^2$

So, first digit of the required value is **2**.

Step 3:

If the last digit of the given number is 1 the last digit of the answer can be 1 or 9.

If the last digit of the given number is 4 the last digit of the answer can be 2 or 8.

If the last digit of the given number is 5 the last digit of the answer should be 5.

If the last digit of the given number is 6 the last digit of the answer can be 4 or 6.

If the last digit of the given number is 9 the last digit of the answer can be 3 or 7.

If the last digit of the given number is 0 the last digit of the answer should be 0.

Square value ends with	Square root ends with
1	1 or 9
4	2 or 8
9	3 or 7
6	4 or 6
5	5
0	0

In 729 the unit digit is 9, so the unit digit of a resultant value is 3 or 7

∴ Required value may be **23 or 27**.

Step 4:

We know that $25^2 = 625$.

Given value is 729, which is greater than 625.

$$\therefore \sqrt{729} = 27$$

Example 2:

Find the square root of 1024.

Solution:

Step 1: 10 _ 24

Step 2:

Perfect square less than 10 is $9 = 3^2$

So, first digit of the required value is 3.

Step 3:

In 1024 the unit digit is 4, so the unit digit of a resultant value is 2 or 8

∴ Required value may be 32 or 38.

Step 4:

We know that $35^2 = 1225$.

Given value is 1024, which is less than 625.

∴ $\sqrt{1024} = 32$

CUBE

Thrice of any number is called as cube of that number.

Cube values of 1 to 30

$1^3 = 1$	$11^3 = 1331$	$21^3 = 9261$
$2^3 = 8$	$12^3 = 1728$	$23^3 = 10648$
$3^3 = 27$	$13^3 = 2197$	$23^3 = 12167$
$4^3 = 64$	$14^3 = 2744$	$24^3 = 13824$
$5^3 = 125$	$15^3 = 3375$	$25^3 = 15625$
$6^3 = 216$	$16^3 = 4096$	$26^3 = 17576$
$7^3 = 343$	$17^3 = 4913$	$27^3 = 19683$
$8^3 = 512$	$18^3 = 5832$	$28^3 = 21952$
$9^3 = 729$	$19^3 = 6859$	$29^3 = 24389$
$10^3 = 1000$	$20^3 = 8000$	$30^3 = 27000$

CUBE ROOT

The cube root of a number is a value that, when multiplied thrice by itself, gives the original number.

Example 1: Find the cube root of 50653.

Step 1: Split the last three digits from the given number and take the remaining digits.

$$50 _ 563$$

Step 2: Find the nearest perfect cube less than the step 1 result.

Take the cube root for that number. That is the first digit of the required value.

i.e., Perfect square less than 50 is $27 = 3^3$

So, first digit of the required value is **3**.

Step 3: The last digit of the given number is last digit of the answer except for 2, 3, 7 and 8.

Cube value ends with	Cube root ends with
0	0
1	1
2	8
3	7
4	4
5	5
6	6
7	3
8	2
9	9

In 50653, the unit digit is 3, so the unit digit of a resultant value is **7**.

∴ Required value is **37**.

Example 2: Find the cube root of 1367631.

Step 1: **1367** _ 631

Step 2: Perfect cube less than **1367** is $1331 = 11^3$

So, first two digit of the required value is 11.

Step 3: In 1367631 the unit digit is 1, so the unit digit of resultant value should be 1.

∴ Required value is 111.

DRILL

I. Find the square value for the following

- 1) 49 2) 33 3) 68 4) 92
5) 572 6) 831 7) 9.7 8) 1.02

II. Find the square root for the following

- 1) $\sqrt{1296}$ 2) $\sqrt{8836}$ 3) $\sqrt{7744}$ 4) $\sqrt{17424}$

III. Find the cube root for the following

- 1) $\sqrt[3]{512}$ 2) $\sqrt[3]{4096}$ 3) $\sqrt[3]{17576}$ 4) $\sqrt[3]{970299}$

IV. Find the cube for the following

- 1) 43 2) 72 3) 59 4) 49

4. POWER AND INDICES

When a number 'P' is multiplied by itself 'n' times, then the product is called nth power of 'P' and is written as P^n . Here, P is called the base and 'n' is known as the index of the power. Therefore, P^n is the exponential expression. P^n is read as 'P raised to the power n or 'P to the power n.

Rules of Indices

If a, b are real numbers ($a, b > 0$) and m, n are real numbers, following properties hold true.

1. $a^m \times a^n = a^{(m+n)}$

2. $a^m \div a^n = a^{(m-n)}$

3. $(a^m)^n = a^{mn}$

4. $(a \times b)^n = a^n \times b^n$

5. $(a \div b)^n = a^n \div b^n$

6. $a^{-m} = \frac{1}{a^m}$

7. $a^0 = 1$

8. $a^{1/2} = \sqrt{a}$;

$$a^{1/m} = \sqrt[m]{a}$$

$$a^{m/n} = (a^{1/n})^m = (\sqrt[n]{a})^m$$

9. If $a^n = b^n$, $a \neq b$; then $n = 0$.

Example: 1

Simplify $(256)^{3/4}$

Solution:

$$(256)^{3/4} = (4^4)^{3/4} = (4)^{4 \times 3/4} = 4^3 = 64$$

Example: 2

Simplify $(1024)^{-3/5}$

Solution:

$$(1024)^{-3/5} = (4^5)^{-3/5} = 4^{[5 \times (-3/5)]} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$$

DRILL

1. $(27)^{2/3} = ?$

a) 3

b) 9

c) 27

d) 6

2. $(\frac{8}{125})^{-4/3} = ?$

a) $\frac{625}{16}$

b) $\frac{16}{625}$

c) $\frac{625}{4}$

d) $\frac{4}{625}$

3. $[(3^2)^6]^4 = 9^x$. Find the value of x
 a) 10 b) 28 c) 24 d) 26
4. $\left[\left\{ \left(\frac{1}{2} \right)^2 \right\}^{-2} \right]^{-1} = ?$
 a) $\frac{1}{16}$ b) $-\frac{1}{16}$ c) 16 d) -16
5. $\left[\left(\sqrt[5]{x^{-\frac{3}{5}}} \right)^{-\frac{5}{3}} \right]^5 = ?$
 a) x^5 b) x^{-5} c) x d) $\frac{1}{x}$
6. $\frac{(43)^{37} \times (43)^{-41}}{(43)^{-6}} = ?$
 a) 3120 b) 1642 c) 2273 d) 1849
7. If $16^9 \times 16^4 \div 16^3 = 16^x$. Find the value of X.
 a) 10 b) 28 c) 24 d) 26
8. $81^{2.5} \times 9^{4.5} \div 3^{4.8} = 9^?$
 a) 7.1 b) 9.4 c) 4.7 d) 4.5
9. $\frac{6^{2/3} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} = ?$
 a) 3 b) 9 c) 7 d) 6
10. If $x = y^a$; $y = z^b$; $z = x^c$; Find the value of abc.
 a) 3 b) 1 c) 2 d) 6
11. If $2^{x-1} + 2^{x+1} = 1280$. Find the value of X.
 a) 10 b) 12 c) 9 d) 8
12. If $3^x - 3^{x-1} = 18$, then x^x is equal to
 a) 3 b) 8 c) 27 d) 216
13. If $5^x = 3125$, then find the value of 5^{x-3} .
 a) 25 b) 54 c) 125 d) 5
14. If $a^{2x+2} = 1$, where a is a positive real number other than 1, then find the value of x.
 a) -2 b) -1 c) 0 d) 1
15. If $\left(\frac{p}{q}\right)^{n-1} = \left(\frac{q}{p}\right)^{n-3}$, then find the value of n.
 a) $\frac{1}{2}$ b) $\frac{7}{2}$ c) 1 d) 2

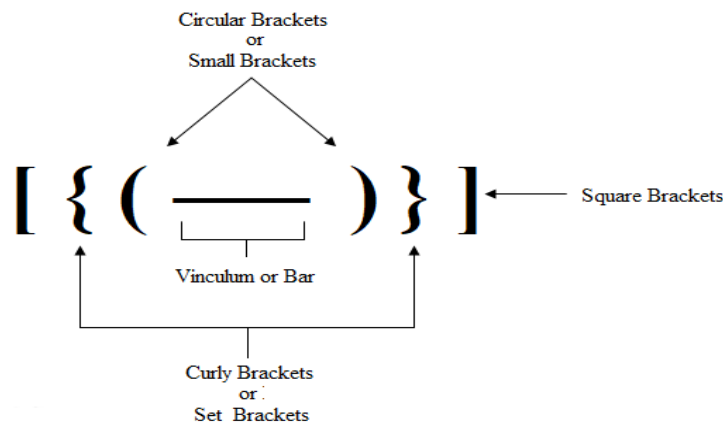
5. SIMPLIFICATION

A complex arithmetical expression can be converted into a simple expression by simplification.

VBODMAS' Rule

To simplify arithmetic expressions, which involve various operations like brackets, multiplication, addition, etc a particular sequence of the operations has to be followed. The operations have to be carried out in the order, in which they appear in the word VBODMAS, where every alphabet have a special stand which are as follows:

1. V represents Vinculum or Bar
2. B represents Bracket
 - First Small brackets (Circular brackets) '()'
 - Second Middle brackets (Curly brackets) '{ }'
 - Third Square brackets (Big brackets) '[]'
3. O represents Of
4. D represents Division
5. M represents Multiplication
6. A represents Addition
7. S represents Subtraction



We should follow the order strictly.

Modulus of a real number x is a positive value. Its modulus is denoted by $|x|$.

Thus, $|5| = 5$ and $|-5| = 5$.

Vinculum of $9-4$: $-\overline{9-4} = -(9-4) = -9 + 4 = -5$

Example 1:

$$\text{Simplify: } 4 - [6 - \{ 12 - (10 - \overline{8-6}) \}]$$

Solution:

$$\begin{aligned} &= 4 - [6 - \{ 12 - (10 - 2) \}] && \text{Remove vinculum} \\ &= 4 - [6 - \{ 12 - 8 \}] && \text{Remove ()} \\ &= 4 - [6 - 4] && \text{Remove \{ \}} \\ &= 4 - 2 && \text{Remove []} \\ &= 2 \end{aligned}$$

Example 2:

Simplify: $(9.6 \times 3.6 \div 7.2 + 10.8 \text{ of } \frac{1}{18} - \frac{1}{10})$

Solution:

$$\begin{aligned} &= 9.6 \times 3.6 \div 7.2 + \mathbf{10.8 \text{ of } \frac{1}{18} - \frac{1}{10}} \\ &= 9.6 \times \mathbf{3.6} \div \mathbf{7.2} + 0.6 - \frac{1}{10} \\ &= \mathbf{9.6} \times \mathbf{0.5} + 0.6 - 0.1 \\ &= \mathbf{4.8} + \mathbf{0.6} - 0.1 \\ &= 5.4 - 0.1 = \mathbf{5.3} \end{aligned}$$

Remove ()

Remove of

Remove ÷

Remove ×

Example 3:

Simplify $18800 \div 940 \div 10$

Solution:

$$\begin{aligned} &= \frac{18800}{940} \div 10 \\ &= 20 \div 10 = \mathbf{2} \end{aligned}$$

Example 4:

Solve $4\frac{1}{6} + 5\frac{1}{6} + 8\frac{1}{6}$

Solution:

$$\begin{aligned} &= 4\frac{1}{6} + 5\frac{1}{6} + 8\frac{1}{6} \\ &= (4 + 5 + 8) \left(\frac{1}{6} + \frac{1}{6} + \frac{1}{6}\right) \\ &= 17 \frac{3}{6} = \mathbf{17\frac{1}{2}} \end{aligned}$$

DRILL

Solve the following expressions

1. $12 + 22 \div 11 \times (27 \div 3)^2 - 10$
2. $37 - [5 + \{28 - (19 - 7)\}]$
3. $19 \times 6 \div 3 + 10 \text{ of } 92 - 81$
4. $\sqrt{15 \times 163 \div 5 - 89}$
5. $(64)^2 \div \sqrt[3]{32768}$
6. $\sqrt{x} + 43 = \sqrt{19881}$
7. $3 + 2 \times 6 - \left(\frac{8}{4} + 6 \times 3\right) + \frac{6}{2}$
8. $9 + 2 \times 14 \div 7 - 7 \left(5 - \frac{18}{6}\right)$
9. $(13 \times 3 \times 3) \div 1.3$
10. $\frac{3}{7} \text{ of } \frac{427}{51} \div 68 \text{ of } \frac{602}{1156} = ?$

Basic Algebraic Formulae for Simplification:

1. $(a + b)^2 = a^2 + b^2 + 2ab$
2. $(a - b)^2 = a^2 + b^2 - 2ab$
3. $(a + b)^2 - (a - b)^2 = 4ab$
4. $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$
5. $(a + b)(a - b) = (a^2 - b^2)$
6. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
7. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
8. $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
9. $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
10. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
11. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

Example 1:

Simplify the following expression

$$\frac{(5.9)^3 + (1.8)^3 + (4.8)^3 - 3 \times 5.9 \times 1.8 \times 4.8}{(5.9)^2 + (1.8)^2 + (4.8)^2 - 5.9 \times 1.8 - 1.8 \times 4.8 - 4.8 \times 5.9}$$

Solution:

We know that,

$$(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\frac{(a^3 + b^3 + c^3 - 3abc)}{(a^2 + b^2 + c^2 - ab - bc - ca)} = (a + b + c)$$

Here $a = 5.9$, $b = 1.8$, $c = 4.8$

$$\frac{(5.9)^3 + (1.8)^3 + (4.8)^3 - 3 \times 5.9 \times 1.8 \times 4.8}{(5.9)^2 + (1.8)^2 + (4.8)^2 - 5.9 \times 1.8 - 1.8 \times 4.8 - 4.8 \times 5.9} = (5.9 + 1.8 + 4.8)$$

$$= 12.5$$

Example 2:

Simplify the following expression

$$\frac{(835 + 378)^2 + (835 - 378)^2}{835 \times 835 + 378 \times 378}$$

Solution:

We know that, $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

$$\frac{(a + b)^2 + (a - b)^2}{a^2 + b^2} = \frac{2(a^2 + b^2)}{a^2 + b^2} = 2$$

Example 3:

If $x = 3 + \sqrt{8}$, then find the value of $x^2 + \frac{1}{x^2}$

Solution:

$$X = 3 + \sqrt{8}$$

Squaring on both sides,

$$X^2 = (3 + \sqrt{8})^2 \quad \text{Since, } (a + b)^2 = a^2 + b^2 + 2ab$$

$$X^2 = 9 + 8 + 6\sqrt{8}$$

$$= 17 + 6\sqrt{8}$$

$$X^2 = 17 + 12\sqrt{2} \text{ ----- (1)}$$

$$\frac{1}{x^2} = \frac{1}{17 + 12\sqrt{2}}$$

Rationalize the denominator,

$$\frac{1}{x^2} = \frac{1}{17 + 12\sqrt{2}} \times \frac{17 - 12\sqrt{2}}{17 - 12\sqrt{2}}$$

$$\frac{1}{x^2} = \frac{17 - 12\sqrt{2}}{289 - 288} \quad \text{Since, } (a + b)(a - b) = (a^2 - b^2)$$

$$\frac{1}{x^2} = 17 - 12\sqrt{2} \text{ ----- (2)}$$

Adding (1) and (2), we get

$$x^2 + \frac{1}{x^2} = (17 + 12\sqrt{2}) + (17 - 12\sqrt{2})$$

$$x^2 + \frac{1}{x^2} = 34$$

DRILL

1. $(578 \times 578 \times 578 + 432 \times 432 \times 432) / (578 \times 578 - 578 \times 432 + 432 \times 432) = ?$

2. $((637 + 478)^2 - (637 - 478)^2) / (637 \times 478) = ?$

3. $((964 + 578)^2 + (964 - 578)^2) / (964 \times 964 + 578 \times 578) = ?$

4. $\frac{(9.8)^3 - (6.8)^3}{9.8^2 + 9.8 \times 6.8 + 6.8^2} = ?$

5. $\frac{(999+588)^2 - (999-588)^2}{999 \times 588} = ?$

6. $\frac{(238+131)^2 + (238-131)^2}{238 \times 238 + 131 \times 131} = ?$

7. $\frac{(0.96)^3 - (0.1)^3}{(0.96)^2 + (0.96) + 0.1} = ?$

8. $387 \times 387 + 113 \times 113 + 2 \times 387 \times 113 = ?$

6. LCM AND HCF

FACTORS AND MULTIPLES

Multiples

Set of numbers which are exactly divisible by the given number. If a number P is exactly divided by Q, we say that P is a multiple of Q.

For example:

Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36.....

Multiples of 12 = 12, 24, 36, 48, 60, 72.....

➤ Multiples of a number are always more than or equal to the given number.

Common Multiple

A common multiple of two or more numbers is a number which is completely divisible (without leaving remainder) by each of them.

For example: The common multiples of 3, 5 and 10 can be obtained as follows

Multiples of 3 = {3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33.....}

Multiples of 5 = {5, 10, 15, 20, 25, 30, 35.....}

Multiples of 10 = {10, 20, 30, 40, 50, 60.....}

∴ Common multiples of 3, 5 and 10 = {30, 60, 90, 120.....}

Least Common Multiple (LCM)

The LCM of two or more given numbers is the least number to be exactly divisible by each of them.

For example: The LCM of 4, 6 and 12 can be obtained as follows

Multiples of 4 = 4, 8, 12, 16, 20, 24, 28, 32, 36.....

Multiples of 6 = 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72.....

Multiples of 12 = 12, 24, 36, 48, 60, 72, 84, 96, 108.....

Common multiples of 4, 6 and 12 = 12, 24, 36, 48, 60, 72.....

∴ LCM of 4, 6 and 12 = 12

Factors

Set of numbers which exactly divides the given number is called as factors. If a number P is exactly divided by Q, we say that Q is a factor of P.

For example:

Factors of 8 = 8, 4, 2, 1

Factors of 12 = 12, 6, 4, 3, 2, 1

Factors of 15 = 15, 5, 3, 1

➤ Factors of a number are always less than or equal to the given number.

Common Factor

A common factor of two or more numbers is that particular number which divides each of them exactly.

For example: The common factors of 12, 48 and 54 can be obtained as follows

Factors of 12 = 12, 6, 4, 3, 2, 1

Factors of 48 = 48, 24, 16, 12, 8, 6, 4, 3, 2, 1

Factors of 54 = 54, 27, 18, 9, 6, 3, 2, 1

∴ Common factors of 12, 48 and 54 = 3, 2, 1

Highest Common Factor (HCF)

HCF of two or more numbers is the greatest number which divides each of them exactly. HCF is also known as Highest Common Divisor (HCD) and Greatest Common Measure (GCM). For example: The HCF of 12 and 18 is 6, as there is no number greater than 6 that divide both 12 and 18. Similarly, the HCF of 6 and 9 is 3.

The common factors of 6, 12 and 18 can be obtained as follows

Factors of 6 = 6, 3, 2, 1

Factors of 12 = 12, 6, 4, 3, 2, 1

Factors of 18 = 18, 9, 6, 3, 2, 1

Common factors of 6, 12 and 18 = 6, 3, 2, 1

∴ Highest Common factors of 6, 12 and 18 = 6

Co-primes

Two numbers are co-primes if their H.C.F is 1.

H.C.F and L.C.M of fractions

➤ $H.C.F = \frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}$

➤ $L.C.M = \frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$

Example 1:

Find the LCM of $\frac{72}{250}$, $\frac{126}{75}$ and $\frac{162}{165}$.

Solution:

Given, $\frac{72}{250} = \frac{36}{125}$, $\frac{126}{75} = \frac{42}{25}$, $\frac{162}{165} = \frac{54}{55}$

$L.C.M = \frac{\text{L.C.M of Numerators}}{\text{H.C.F of Denominators}}$

LCM of 36, 42, 54 = 756

HCF of 125, 25, 55 = 5

$LCM \left(\frac{72}{250}, \frac{126}{75} \text{ and } \frac{162}{165} \right) = \frac{756}{5} = 151 \frac{1}{5}$

Example 2:

Find the HCF of $\frac{36}{51}$ and $3\frac{9}{17}$.

Solution:

Given, $\frac{36}{51} = \frac{12}{17}$, $3\frac{9}{17} = \frac{60}{17}$

$$\text{H.C.F} = \frac{\text{H.C.F of Numerators}}{\text{L.C.M of Denominators}}$$

HCF of 12, 60 = 12

LCM of 17, 17 = 17

HCF ($\frac{36}{51}$, $3\frac{9}{17}$) = $\frac{12}{17}$

Example 3:

Seven bells ring at intervals of 2, 3, 4, 6, 8, 9 and 12 minutes, respectively. They started ringing simultaneously at 5:00 in the morning. What will be the next time when they all ring simultaneously?

Solution:

LCM of 2, 3, 4, 6, 8, 9 and 12 mins = 72 min = 1 h 12 min

∴ Required time = (5 + 1:12) = 6:12 in the morning.

Example 4:

Six bells ring at intervals of 2, 4, 6, 8, 10 and 12 seconds, respectively. They started ringing simultaneously. How many times, will they ring together in 30 min?

Solution:

LCM of 2, 4, 6, 8, 10 and 12 seconds = 120 seconds = 2 minutes

i.e., Bells will ring together for every 2 minutes

∴ Required number of times = ($\frac{30}{2} + 1$) = 16 times

DRILL

1. Find the LCM and HCF of 8, 12 and 36.

a) 72, 4	b) 72, 6	c) 76, 8	d) 76, 4
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2. Find the LCM and HCF of 72, 108 and 24.

a) 216, 4	b) 108, 12	c) 216, 12	d) 108, 6
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3. Find the LCM and HCF of 125, 100 and 25.

a) 500, 5	b) 600, 5	c) 500, 25	d) 250, 25
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4. Find the HCF of 132, 204 and 228.

a) 12	b) 18	c) 6	d) 21
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5. Compute HCF of $(2^2 * 2^3 * 5^1 * 7^4)$, $(2^3 * 3^2 * 5^2 * 7^3)$ and $(2^2 * 5^3 * 7^5)$.

a) 6760	b) 6860	c) 6960	d) 7060
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6. Find the LCM and HCF of $\frac{1}{3}$, $\frac{5}{6}$, $\frac{2}{9}$ and $\frac{4}{27}$.
- a) $\frac{20}{9}, \frac{1}{27}$ b) $\frac{20}{3}, \frac{1}{3}$ c) $\frac{20}{3}, \frac{1}{54}$ d) $\frac{20}{6}, \frac{1}{9}$
7. Find the HCF and LCM of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{10}{27}$ and $\frac{64}{81}$.
- a) $\frac{2}{9}, \frac{320}{3}$ b) $\frac{2}{81}, \frac{320}{3}$ c) $\frac{2}{9}, \frac{320}{9}$ d) $\frac{2}{81}, \frac{320}{9}$
8. If three numbers are $2a$, $5a$ and $7a$, what will be their LCM and HCF?
- a) $70a$, a b) $65a$, a^2 c) $75a$ d) $70a^3$, a^3
9. Product of two co-prime numbers is 117. Then, their LCM is
- a) 9 b) 13 c) 39 d) 117
10. Six bells rings at an intervals of 1s, 3s, 4s, 6s, 9s and 12s respectively. They started ringing simultaneously at noon. When will they ring simultaneously next time?
- a) 12:00:36 am b) 12:00:36 pm c) 12:00:12 pm d) 12:00:12 am
11. Four bells rings at an intervals of 2s, 4s, 8s and 10s respectively. They started ringing simultaneously. How many times will they ring together in 20 mins?
- a) 30 b) 31 c) 10 d) 11
12. The length and breadth of the rectangular sheet are 12 cm and 8 cm respectively. Maximum sizes of the squares to be cut without wasting the sheet. Find the sides of squares and also find the possible number of squares can be formed.
- a) 4 cm, 8 b) 6 cm, 4 c) 4 cm, 6 d) 8 cm, 6
13. If 30 tennis balls and 15 rockets to be distributed equally among students, find the maximum possible number of students.
- a) 15 b) 30 c) 10 d) 5

PRACTISE QUESTIONS

1. Find the LCM and HCF of 24, 40 and 36.
- a) 720, 4 b) 720, 6 c) 360, 8 d) 360, 4
2. Find the LCM and HCF of 16, 12 and 36.
- a) 144, 4 b) 120, 8 c) 144, 8 d) 120, 4
3. Find the LCM and HCF of $\frac{9}{10}$, $\frac{12}{25}$, $\frac{18}{35}$ and $\frac{21}{40}$.
- a) $\frac{252}{5}, \frac{3}{2800}$ b) $\frac{252}{5}, \frac{3}{1400}$ c) $\frac{252}{25}, \frac{3}{2800}$ d) $\frac{126}{5}, \frac{3}{1400}$
4. Find the LCM and HCF of $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$ and $\frac{9}{13}$.
- a) $\frac{36}{1}, \frac{1}{1365}$ b) $\frac{12}{1}, \frac{1}{685}$ c) $\frac{36}{1}, \frac{1}{685}$ d) $\frac{36}{1}, \frac{1}{560}$
5. Clock A rings for every 5 hours, Clock B for every 3 hours and Clock C for every hour. If they ring together at 12 noon, when will they ring together next time?
- a) 3 pm b) 3 am c) 2 am d) 2 pm
6. Four electronic devices make a beep after duration of 30 minutes, 1 hour, $\frac{3}{2}$ hours and 1 hour 45 min respectively. If all the devices beeped together at 12 noon at what time will they beep together again?
- a) 9 am b) 10 am c) 11 am d) 11:30 am

7. Three bells chime at intervals of 48, 60 and 90 min, respectively. If all the three bells chime together at 10:00 am, at what time will all three chime again that day?
 a) 1:00 pm b) 2:00 pm c) 8:00 pm d) 10:00 pm
8. Five bells begin to toll together at intervals of 9 s, 6 s, 4 s, 10 s and 8 s, respectively. How many times will they toll together in the span of 1 h (excluding the toll at the start)?
 a) 5 b) 8 c) 10 d) can't be determined
9. Three pieces of timber 84 m, 98 m and 126 m long have to be divided into planks of the same length. What is the greatest possible length of each plank?
 a) 14 m b) 28 m c) 7 m d) 21 m

OTHER IMPORTANT CONCEPTS

- Product of two numbers = Product of their H.C.F and L.C.M
- When the HCF of each pair of n given numbers is x and their LCM is y, then
 Product of these numbers = $(\text{HCF})^{n-1} \times \text{LCM} = (x)^{n-1} (y)$
- The least number which when divided by x, y and z leaves the same remainder k in each case, is given by LCM of (x, y, z) + k.
- The least number which when divided by x, y and z leaves the remainders a, b and c respectively, is given by LCM of (x, y, z) – k, where, $k = (x - a) = (y - b) = (z - c)$.
- The greatest number that will divide x, y and z..... leaving the same remainder in each case, is given by HCF of (|x- y| , |y-z| , |z-x|)....
- The greatest number that will divide x, y and z, leaving the remainders a, b, c respectively is given by HCF of (x – a) , (y – b) , (z – c).
- The greatest n-digit number which when divided by x, y and z leaves no remainder, then
 Required number = n-digit greatest number – R
- The greatest n-digit number which when divided by x, y and z leaves remainder k, then
 Required number = [n-digit greatest number – R] + k
 Where, R is the remainder obtained when n-digit greatest number is divided by the LCM of x, y and z.
- The smallest n-digit number which when divided by x, y and z leaves no remainder, then
 Required number = [n-digit smallest number + (L – R)]
- The smallest n-digit number which when divided by x, y and z leaves remainder k, then
 Required number = [n-digit smallest number + (L – R)] + k
 Where, R is the remainder obtained when n-digit smallest number is divided by the LCM of x, y and z. L is the LCM of x, y and z.

Example 1:

The LCM of 2 numbers is 2079 and their HCF is 27. If the 1st number is 189, then find the second number.

Solution:

Here, LCM = 2079, HCF = 27, 1st number = 189, 2nd number = ?

$$\text{Product of two numbers} = \text{HCF} \times \text{LCM}$$

$$189 \times 2^{\text{nd}} \text{ number} = 27 \times 2079$$

$$2^{\text{nd}} \text{ number} = \frac{27 \times 2079}{189}$$

$$= 319$$

Example 2:

There are five numbers. HCF of each possible pair is 4 and LCM of all the five numbers is 27720. What will be the product of all the five numbers?

Solution:

Given, HCF = 4, LCM = 27720 and $n = 5$

According to the formula, Required product = $(\text{HCF})^{n-1} \times \text{LCM}$

$$= (4)^{5-1} \times 27720$$

$$= (4)^4 \times 27720$$

$$= 256 \times 27720$$

$$= 7096320$$

Example 3:

Find least number which when divided by 24, 30 and 54 leaves 5 as remainder in each case.

Solution:

The least number which when divided by x , y and z leaves the same remainder k in each case, is given by **LCM of $(x, y, z) + k$.**

Given that, $x = 24$, $y = 30$, $z = 54$ and $k = 5$

Required number = [LCM of (24, 30 and 54)] + 5

$$\text{LCM of } 24, 30, 54 = 1080$$

\therefore Required number = 1080 + 5 = 1085

Example 4:

Find least number which when divided by 24, 32 and 36 leaves the remainders 19, 27 and 31 respectively.

Solution:

The least number which when divided by x , y and z leaves the remainders a , b and c respectively, is given by **LCM of $(x, y, z) - k$** .

$$\text{Where, } k = (x - a) = (y - b) = (z - c)$$

Given that, $x = 24, y = 32, z = 36,$

$$a = 19, b = 27 \text{ and } c = 31$$

Here, $k = (24 - 19) = (32 - 27) = (36 - 31) = 5$

$$\text{Common difference} = 5$$

Required number = (LCM of 24, 32 and 36) - 5

$$\text{Now, LCM of 24, 32 and 36} = 288$$

$$\therefore \text{Required number} = 288 - 5 = 283$$

Example 5:

What is the greatest number that will divide 99, 123 and 183 leaving the same remainder in each case? Also, find the common remainder.

Solution:

The greatest number that will divide x , y and z leaving the same remainder in each case, is given by **HCF of $(|x - y|, |y - z|, |z - x|)$**

Given that, $x = 99, y = 123$ and $z = 183$

$$\text{Required number} = \text{HCF of } (|x - y|, |y - z|, |z - x|)$$

$$|x - y| = |99 - 123| = 24$$

$$|y - z| = |123 - 183| = 60$$

$$|z - x| = |183 - 99| = 84$$

Therefore, $\text{HCF}(24, 60, 84) = 12$

So, the greatest number that divides 99, 123 and 183 is 12.

To find the common remainder

$$\frac{99}{12} = 8 \frac{3}{12}, \frac{123}{12} = 10 \frac{3}{12}, \frac{183}{12} = 15 \frac{3}{12}$$

$$\therefore \text{Required common remainder} = 3$$

Example 6:

Find the greatest number which divides 29, 60 and 103 leaving remainders 5, 12 and 7 respectively.

Solution:

The greatest number that will divide x, y and z, leaving the remainders a, b, c respectively is given by **HCF of (x – a) , (y – b) , (z – c)**.

Given that,

$$x = 29, y = 60, z = 103$$

$$a = 5, b = 12 \text{ and } c = 7$$

$$\begin{aligned} \text{Required number} &= \text{HCF of } [(29 - 5), (60 - 12), (103 - 7)] \\ &= \text{HCF of } 24, 48, 96 \end{aligned}$$

$$\therefore \text{Required HCF of } 24, 48 \text{ and } 96 = 24$$

Hence, 24 is the required number.

Example 7:

Find the largest possible number of 5 digits which is exactly divisible by 32, 36 and 40.

Solution:

The greatest n-digit number which when divided by x, y and z leaves no remainder, then

$$\text{Required number} = \text{n-digit greatest number} - \mathbf{R}$$

Where, R is remainder obtained when n-digit greatest number is divided by LCM of x, y and z.

Given numbers are 32, 36 and 40

$$\text{LCM of } 32, 36 \text{ and } 40 = 1440$$

$$\text{Greatest 5-digit number} = 99999$$

On dividing 99999 by 1440, we get

$$\frac{99999}{1440} = 69\frac{639}{1440}$$

$$\therefore \text{Required number} = 99999 - 639 = 99360$$

Example 8:

Find the least possible 5-digit number, which when divided by 10, 12, 16 and 18 leaves remainder 27.

Solution:

The smallest n-digit number which when divided by x, y and z leaves remainder k, then

$$\text{Required number} = [\text{n-digit smallest number} + (\mathbf{L} - \mathbf{R})] + \mathbf{k}$$

$$\text{LCM of } 10, 12, 16 \text{ and } 18 = 720$$

$$\text{Smallest 5-digit number} = 10000$$

On dividing 10000 by 720, we get

$$\frac{10000}{720} = 13\frac{640}{720}$$

$$\therefore \text{Required number} = 10000 + (720 - 640) + 27 = 10000 + 80 + 27 = 10107$$

DRILL

1. The LCM of two numbers is 2376 while their HCF is 33. If one of the number is 297, then find the other number.
a) 216 b) 264 c) 642 d) 792
2. The ratio of two numbers is 3 : 4 and their HCF is 4. What will be their LCM?
a) 12 b) 16 c) 24 d) 48
3. The ratio of two numbers is 5 : 6 and their LCM is 480, Find their HCF.
a) 20 b) 16 c) 6 d) 5
4. The HCF of three numbers is 23. If they are in the ratio of 1: 2: 3, then find the numbers.
a) 69, 15, 22 b) 23, 46, 69 c) 25, 31, 41 d) 23, 21, 35
5. Three numbers are in the ratio of 3 : 4 : 5 and their LCM is 1200. Find HCF of numbers.
a) 40 b) 30 c) 80 d) 20
6. What is the least number which is exactly divisible by 8, 9, 12, 15 and 18 and is also a perfect square?
a) 3600 b) 7200 c) 5200 d) 6500
7. Find the greatest number of 3-digits which when divided by 6, 9 and 12 leaves 3 as remainder in each case.
a) 975 b) 996 c) 903 d) 939
8. What will be the greatest number that divides 1356, 1868 and 2764 leaving 12 as remainder in each case?
a) 64 b) 124 c) 156 d) 260
9. Find the greatest number that divides 130, 305 and 245 leaving remainders 6, 9 and 17 respectively?
a) 4 b) 5 c) 14 d) 24
10. Find the largest number which divides 1305, 4665 and 6905 leaving same remainder in each case. Also, find the common remainder.
a) 1210, 158 b) 1120, 158 c) 1120, 185 d) 1210, 185
11. What will be the least number which when divided by 12, 21 and 35 leaves 6 as remainder in each case?
a) 426 b) 326 c) 536 d) 436
12. Find the least number which when divided by 18, 24, 32 leaves the remainder 4, 10 and 18 respectively.
a) 274 b) 218 c) 284 d) 270
13. Find least number which when diminished by 7, is divisible by each one of 21, 28, 36, 45.
a) 1255 b) 1177 c) 1265 d) 1267
14. What is the least number which when increased by 9, is divisible by each one of 24, 32, 36 and 54?
a) 855 b) 890 c) 756 d) 895

7. SIMPLE EQUATION

DRILL

1. If $2x + 4 = 14$, then find x .
2. Find the value of x & y from the given equations. $2x + 3y = 17$; $3x - 2y = 6$
3. Sum of three consecutive numbers is 45. Find the numbers.
4. 12 less than twice the number is 32. Find the number.
5. If cost of 7 books is Rs.245, then find the cost of one book.
6. In a class of 50 students, the number of girls is $\frac{2}{3}$ of the boys. Find the number of boys & girls.
7. Total cost of 5 CDs & 2 floppies is Rs.95. Cost of one CD is Rs.5 more than 1 floppy. Find the cost of one CD.
8. Smith has some 2 rupee cons and 5 rupee coins. If the total number of coins is 15 and the total amount is Rs.51, then find the number of coins in each category.
9. The LCM of two numbers is 45 times of their HCF. If one of the numbers is 125 & sum of their LCM and HCF is 1150, then find the other number.

PRACTISE QUESTIONS

1. The present age of Chitra is 23 years. She is 25 years younger to her mother. Find her mother's age.
2. The breadth of the rectangle is 5 cm less than its length. If the perimeter of the rectangle is 38 cm, then find its length & breadth.
3. The cost of 3 mangoes & 4 oranges is Rs.90 and cost of 4 mangoes & 3 oranges is Rs.85. Find the cost of a mango & an orange.
4. Sum of 4 consecutive even numbers is 220. Find the numbers.
5. Sum of 2 numbers is 18 & twice the difference between them is 4. Find the numbers.
6. Some sweets were to be distributed equally among 540 students of a school. But due to the absence of 120 students each child got 4 more sweets. How many sweets were to be distributed to each student originally?

8. RATIO AND PROPORTION

RATIO

A ratio is a comparison of two or more quantities in the same unit. The following notations express the ratio of x to y .

$$\Rightarrow x : y \text{ or } x/y$$

In the ratio $x : y$, x is the first term or the antecedent and y is the second term or the consequent.

Basic rules

- ✓ Ratio should be a whole number.
- ✓ It should always be in simplified form.
- ✓ If ratio in the form of fraction, simplify it by taking LCM.

Examples

- If A is four times of B , then $A : B = 4 : 1$
- If A is four times more than B , then $A : B = 5 : 1$

PROPORTION

The equality of two ratios is called proportion. If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a , b , c and d are in proportion. Here, the first and fourth terms are known as extremes, while the second and third term are known as means.

$$\Rightarrow \text{Product of mean} = \text{Product of extremes}$$

DRILL

1. If $a:b=7:9$ and $b:c=9:11$, then find $a:c$.
2. If $a:b=3:4$ and $b:c=12:7$, then find $a:b:c$.
3. If $a:b=1:2$ and $b:c=3:4$ and $c:d=5:6$, then find $a:d$.
4. If $2a=3b=4c$, then find $a:b:c$.
5. If $a/2 = b/3 = c/4$, then find $a : b : c$.
6. If $a : b = 1/4 : 1/3$ and $b : c = 1/12 : 1/2$, then find $a:b:c$.
7. A 's share is twice that of B 's share. Three times of B 's share is equal to four times of C 's share. Find $A:B:C$.
8. Find the 4th proportional to 4,9,12.
9. Find the third proportional to 16 and 36.
10. Find the mean proportional between 16&9.
11. If Rs.700 is divided between A & B in the ratio 3:4, then find A 's share.

12. The ratio of 2 numbers is 5:6 and their sum is 77. Find the difference of the numbers.
13. The ratio of two numbers is 10:7 and their difference is 45. Find the sum of the numbers.
14. The ratio of three numbers is 3:4:7 and their product is 18144. Find the numbers.
15. Ratio between 2 numbers is 3:4 and sum of their square is 625. Find the numbers.
16. Ratio between 2 numbers is 5:3 and difference between their squares is 144. Find the numbers.
17. The ratio between male population and female population of a village is 17:13. If the number of male is 280 more than that of the female, then find the total population of that village?
18. Two numbers are in the ratio of 5:6. If 7 is added to each number, then the ratio becomes 6:7. Find the original numbers.
19. The first number is 5 times the second number. If 3 is subtracted from each number then the first number becomes 7 times the second number. Find the original numbers.
20. Two numbers are in the ratio 3:5. If 2 is added to the first number and 5 is added to the second number, then the ratio becomes 4:7. Find the smaller number.
21. 132 liters of a mixture contains milk and water in the ratio 8:3. How much water must be added to the mixture so as to make the ratio of milk and water 2:1?
22. Some quantity of rice is required for each member of a family of 15 members. On a particular day due to the absence of some members, the consumption of rice was reduced to the ratio 5:3. Find the number of members absent on that day.

PRACTICE QUESTIONS

1. Find the ratio between $2^{2.5}$ and $2^{0.5}$
2. A certain amount of money is distributed among A, B and C. A gets $\frac{3}{16}$ th and B gets $\frac{1}{4}$ th of whole amount. If C gets Rs.81, then find B's share on that amount.
3. Divide Rs.1570 between A and B, so that after Rs.25 being deducted from A's share and Rs.45 from B's share, their shares are in the ratio 2:3. Find the amount received by A.
4. Two numbers are in the ratio 8:11. If we subtract 6 from each number, the new ratio becomes 7:10. Find the original numbers.
5. A vessel contains milk and water in the ratio 5:7. Had it contained 2 litres more milk and 2 litres less of water, the ratio should become 4:5. Find the quantity of milk in the vessel.
6. A boy read $\frac{3}{8}$ of a book on one day and $\frac{4}{5}$ of the book on another day. If there were 30 pages unread, how many pages did the book contain?

9. PROBLEMS ON AGES

Problems based on ages generally consist of information ages of two or more persons and a relationship between their ages in present / future / past.

DRILL

1. The sum of ages of father and his daughter is 54 years. After 6 years the age of the father will be double the age of the daughter. What is the age of the daughter?
2. The difference between the present age of Arun and Deepak is 14 years. Seven years ago the ratio of their ages was 5:7 respectively. What is Deepak's present age?
3. The average age of a man and his daughter is 19 years. The ratio of their ages is 16:3. What is the difference between the ages of the man and daughter after 6 years?
4. Four years ago Shyam's age was $\frac{3}{4}$ times that of Ram. Four year hence, Shyam's age will be $\frac{5}{6}$ times that of Ram. What is the present age of Shyam?
5. The ratio of present age of Sita and her son is 5:2 respectively. Seven years hence the ratio will be 2:1. What was the age of Sita when her son was born?
6. Father is aged three times more than his son Rohit. After 8 years, he would be two and a half times of Rohit's age. After further 8 years, how many times would he be of Rohit's age?
7. The ratio between the present age of A and B is 7:5 respectively. The ratio between A's age 5 years ago and B's age 5 years hence is 1:1. What is the ratio between A's age 5 years hence and B's age 5 years ago?
8. Raju got married 9 years ago. Today his age is $1\frac{1}{3}$ times of his age at the time of his marriage. Find his present age.
9. 3 Years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average age of the family is the same today. What is the present age of the baby?
10. The ratio of the ages of a husband and his wife is 5:4. After 12 years this ratio will be 7:6. If at the time of their marriage the ratio was 9:7, then how many years ago they got married?

PRACTICE QUESTIONS

1. Three years ago the age of a father was 7 times the age of his son. At present the father's age is 5 times that of his son. What is the present age of the father and his son?
2. The present age of Ram's father is four times of Ram's age. Five years from now, the father's age will be three times the age of Ram. What is the present age of Ram's father?
3. The age of a man after 15 years is 4 times the age of that man 15 years before. Find his present age.
4. In 30 years, age of a person becomes 3 times of his present age. What is his present age?
5. If Akshay is as much elder than Vinay as he is younger to Karthik and sum of ages of Vinay and Karthik is 48 yrs, then find the age of Akshay.
6. In a family, the average age of father and mother is 35 yrs. The average age of the father, mother and their only son is 27 yrs. What is the age of the son?
7. In 5 years A will be twice as old as B was 5 years ago. If A is now 11 years older than B, then find the present age of B?
8. The present age of a father is 3 years more than three times the age of his son. Three years hence the father's age will be 10 years more than twice the age of the son. Find the present age of the father.
9. The ratio of father's age to his son's age is 7:3. If the product of their ages is 756, then find the ratio of their ages after 6 year.
10. Father is four times the age of his daughter. If after 5 years, he would be three times of daughter's age, then further after 5 years, how many times more he would be of his daughter's age?
11. The ratio of Akash & Vinesh age is 9:7. If the difference between the present age of Akash & the age of Vinesh 5 years hence is 3, then find the sum of their present ages?
12. The total age of A and B is 12 years more than the total age of B and C. C is how many year younger than A.
13. Ratio of ages of Prasad and Pavan is 10 : X. Prasad is 18 years younger to Vinod. After nine years Vinod will be 47 years old. If difference between ages of Prasad and Pavan is same as age of Vinod, find the value of X?

10. PARTNERSHIP

When two or more persons make an association and invest money for running a certain business and after certain time receive profit in the ratio of their invested money and time period of investment, then such an association is called partnership and the persons involved in the partnership are called partners.

BASIC CONCEPTS

- ✓ If the Investment period is Same for all the Partners

$$\text{Profit Ratio} = \text{Capital Ratio}$$

- ✓ If the Investment period is different among the Partners

$$\text{Profit Ratio} = \text{Capital Ratio} \times \text{Time Ratio}$$

DRILL

1. Ram, Raj, and Ravi together started a business in partnership. The Ratio of their capital was 3:4:7. If their annual profit was Rs.2100, what would be the share of Ravi in the profit?
2. P, Q and R start a business jointly. Twice the capital of P is equal to thrice the capital of Q and the capital of Q is four times the capital of R. Find the share of Q in an annual profit of Rs.148500.
3. A, B, C purchase a farm for Rs.10000 in which A pays Rs.4000. They sell it so as to gain a certain sum, in which B gets Rs. 275 and C gets Rs. 175. Find the share of A in the profit?
4. A, B, C started a business with investment of Rs.5000 each. A remains in partnership for 9 months, B for 6 months and C for 12 months. Find the ratio of their profit.
5. A & B started a firm together. A's investment was thrice the investment of B and the period of his investment was two times the period of investment of B. If B received Rs.4000 as profit, then find their total profit.
6. A and B started a business by investing Rs.12,000 and Rs.16,000 respectively. C joined them with Rs.15000 after 8 months. Find C's share in total profit of Rs.45,600 earned at the end of 2 years.
7. A & B started a business by investing Rs.20000 and Rs.25000, respectively. After 4 months, B left and C joined by investing Rs.15000. At the end of the year, there was a profit of Rs.23000. What was C's share?
8. A, B and C invest Rs.50,000 in a business. A invests Rs.4000 more than B and B invests Rs.5000 more than C. Out of a total profit of Rs.70,000, what is the share received by A?

9. Arun started a business with 4500 and Kumar joined afterwards with Rs.3000. How long Kumar's capital was used in the business, if the profit at the end of the year were divided in the ratio 2:1?
10. In a partnership, A invests $\frac{1}{3}$ of the capital for $\frac{1}{4}$ of the time. B invests $\frac{1}{5}$ of the capital for $\frac{1}{3}$ of the time and C invests rest of the capital for whole time. Find A's share out of the total profit of Rs.3737 at the end of a year.
11. Three partners shared the profit in a business in the ratio 5:7:8. They had partnered for 14 months, 8 months and 7 months respectively. What was the ratio of their investment?
12. A and B entered into a partnership investing Rs.16000 and Rs.12000 respectively. After 3 months, A withdrew Rs.5000 while B invested Rs.5000 more. After 3 more months C joined the business with a capital of Rs.21000. If at the end one year they earned a profit of Rs.13200, find the value did the share of B exceeded that of C.

PRACTICE QUESTIONS

1. A, B and C entered into partnership in a business. A got $\frac{3}{5}$ of the profit. B and C distributed the remaining profit equally. If C got Rs.400 less than A, find the total profit.
2. A starts a business with Rs.2000 and B joins him after 3 months with Rs.8000. Find the ratio of their profit at the end of the year?
3. Karan started a business investing Rs.18000. After four months, Ram joined with a capital of Rs.24000. If at the end of the year, they earned a profit of Rs.5100, then what would be the share of Ram in the profit?
4. A & B enter into a partnership with capital in the ratio of 7:8. At the end of 10 months, A withdraws. If they receive the profit in the ratio of 7:9, how long B's capital is used?
5. X and Y make a partnership. X invests Rs.8000 for 8 months and Y remains in the business for 4 months. Out of the total profit, Y claims $\frac{2}{7}$ of the profit. How much money is contributed by Y?
6. A, B and C enter into a partnership and their shares are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. After 2 months, A withdraws half of his capital and after 10 months, a profit of Rs. 378 is divided among them. What is B's share?
7. A, B and C invested capitals in the ratio of 2:3:5. At the end of the business terms, they received the profit in the ratio of 5:3:12. Find the ratio of time for which they contributed their capital.
8. A & B invest in a business in the ratio 3:2. If 5% of the total profit goes to charity and A's share is Rs.855, then what is the total profit?

11. PERCENTAGE

The term per cent means 'for every hundred'. It can be defined as "A per cent is a fraction whose denominator is 100 and the numerator of the fraction is called the rate per cent." Percentage is denoted by the sign '% '.

Conversion of Percentage into Fraction

$$\text{Convert } x\% \text{ percentage into fraction} = \frac{x}{100}$$

Conversion of Fraction into Percentage

$$\text{Convert } x/y \text{ Fraction into Percentage} = \frac{x}{y} \times 100$$

RELATIONSHIP BETWEEN FRACTION AND PERCENTAGE:

$1 = \frac{1}{1} = 100 \%$ $2 = \frac{2}{1} = 200 \%$ $3 = \frac{3}{1} = 300 \%$	$\frac{1}{4} = 25 \%$ $\frac{2}{4} = 50 \%$ $\frac{3}{4} = 75 \%$	$\frac{1}{6} = 16.67 \%$ $\frac{2}{6} = 33.33 \%$ $\frac{3}{6} = 50 \%$	$\frac{6}{7} = 87.71 \%$ $\frac{7}{7} = 100 \%$ $\frac{8}{7} = 114.28 \%$
$\frac{1}{2} = 50 \%$ $\frac{2}{2} = 100 \%$ $\frac{3}{2} = 150 \%$ $\frac{4}{2} = 200 \%$	$\frac{4}{4} = 100 \%$ $\frac{5}{4} = 125 \%$ $\frac{1}{5} = 20 \%$ $\frac{2}{5} = 40 \%$	$\frac{4}{6} = 66.67 \%$ $\frac{5}{6} = 83.33 \%$ $\frac{6}{6} = 100 \%$ $\frac{7}{6} = 116.67 \%$	$\frac{1}{8} = 12.5 \%$ $\frac{2}{8} = 25 \%$ $\frac{3}{8} = 37.5 \%$ $\frac{4}{8} = 50 \%$
$\frac{1}{3} = 33.33 \%$ $\frac{2}{3} = 66.67 \%$ $\frac{3}{3} = 100 \%$ $\frac{4}{3} = 133.33 \%$ $\frac{5}{3} = 166.67 \%$	$\frac{3}{5} = 60 \%$ $\frac{4}{5} = 80 \%$ $\frac{5}{5} = 100 \%$ $\frac{6}{5} = 120 \%$ $\frac{7}{5} = 140 \%$	$\frac{1}{7} = 14.28 \%$ $\frac{2}{7} = 28.57 \%$ $\frac{3}{7} = 42.85 \%$ $\frac{4}{7} = 57.14 \%$ $\frac{5}{7} = 71.42 \%$	$\frac{5}{8} = 62.5 \%$ $\frac{6}{8} = 75 \%$ $\frac{7}{8} = 87.5 \%$ $\frac{8}{8} = 100 \%$ $\frac{9}{8} = 112.5 \%$

$\frac{1}{9} = 11.11\%$
$\frac{2}{9} = 22.22\%$
$\frac{3}{9} = 33.33\%$
$\frac{4}{9} = 44.44\%$
$\frac{5}{9} = 55.55\%$
$\frac{6}{9} = 66.66\%$
$\frac{7}{9} = 77.77\%$
$\frac{8}{9} = 88.88\%$
$\frac{9}{9} = 100\%$
$\frac{10}{9} = 111.11\%$
$\frac{1}{10} = 10\%$
$\frac{2}{10} = 20\%$
$\frac{3}{10} = 30\%$
$\frac{4}{10} = 40\%$
$\frac{5}{10} = 50\%$
$\frac{6}{10} = 60\%$
$\frac{7}{10} = 70\%$
$\frac{8}{10} = 80\%$
$\frac{9}{10} = 90\%$
$\frac{10}{10} = 100\%$

$\frac{1}{11} = 9.09\%$
$\frac{2}{11} = 18.18\%$
$\frac{3}{11} = 27.27\%$
$\frac{4}{11} = 36.36\%$
$\frac{5}{11} = 45.45\%$
$\frac{6}{11} = 54.54\%$
$\frac{7}{11} = 63.63\%$
$\frac{8}{11} = 72.72\%$
$\frac{9}{11} = 81.81\%$
$\frac{10}{11} = 90.9\%$
$\frac{11}{11} = 100\%$
$\frac{12}{11} = 109.09\%$
$\frac{13}{11} = 118.18\%$
$\frac{1}{12} = 8.33\%$
$\frac{2}{12} = 16.67\%$
$\frac{3}{12} = 25\%$
$\frac{4}{12} = 33.33\%$
$\frac{5}{12} = 41.67\%$
$\frac{6}{12} = 50\%$
$\frac{7}{12} = 58.33\%$

$\frac{8}{12} = 66.67\%$
$\frac{9}{12} = 75\%$
$\frac{10}{12} = 83.33\%$
$\frac{11}{12} = 91.67\%$
$\frac{12}{12} = 100\%$
$\frac{13}{12} = 108.33\%$
$\frac{1}{15} = 6.67\%$
$\frac{2}{15} = 13.33\%$
$\frac{3}{15} = 20\%$
$\frac{4}{15} = 26.67\%$
$\frac{5}{15} = 33.33\%$
$\frac{6}{15} = 40\%$
$\frac{1}{16} = 6.25\%$
$\frac{2}{16} = 12.5\%$
$\frac{3}{16} = 18.75\%$
$\frac{4}{16} = 25\%$
$\frac{1}{20} = 5\%$
$\frac{2}{20} = 10\%$
$\frac{3}{20} = 15\%$
$\frac{4}{20} = 20\%$

$\frac{1}{24} = 4.167\%$
$\frac{2}{24} = 8.33\%$
$\frac{3}{24} = 12.5\%$
$\frac{4}{24} = 16.667\%$
$\frac{5}{24} = 20.833\%$
$\frac{6}{24} = 25\%$
$\frac{7}{24} = 29.167\%$
$\frac{8}{24} = 33.33\%$
$\frac{9}{24} = 37.5\%$
$\frac{10}{24} = 41.67\%$
$\frac{1}{25} = 4\%$
$\frac{2}{25} = 8\%$
$\frac{3}{25} = 12\%$
$\frac{4}{25} = 16\%$
$\frac{1}{32} = 3.125\%$
$\frac{2}{32} = 6.25\%$
$\frac{1}{50} = 2\%$
$\frac{2}{50} = 4\%$
$\frac{1}{100} = 1\%$

DRILL**Numeric Problem**

1. Express each of the following as a fraction
 i) 56% ii) 4% iii) 0.6% iv) 0.08%
2. 18% of 450 = x, find x.
3. x % of 50=2, find x.
4. 45% of x = 72, find x.
5. 20% of x is equal to 30% of 40.Find x value.
6. 14% of 14 = x, find x.

Arithmetic Problems

1. A number when increased by 25% gives 80.Find the number.
2. If A's income is 40% less than that of B, then
 - i) Find the percentage of A with respect to B.
 - ii) What is the percentage of B to that of A?
 - iii) How much percent B's income is more than that of A?
 - iv) How much percent A's income is less than that of B?
3. If the numerator is increased by 200% and the denominator is increased by 300%, the resultant fraction is $\frac{3}{8}$. What is the original fraction?
4. In an examination a candidate has to secure 40% of the marks to pass. If a candidate secures 220 marks and fails by 20 marks, find the total marks of the examination.
5. If Ram's salary was decreased by 50% and subsequently increased by 50%, how much percent did he lose?
6. Tamil spends 25% of his monthly income on rent, 30% on food and 20% on other. If he saves Rs.5000 per month, what is his monthly income?
7. A person spends 20% of his monthly income on food and 15 % of the remaining on cloths and saves the remaining amount. If he saves Rs.121176 per annum, then what amount does the person spend on food per month?

8. In an election there were only two candidates. One of the candidates secured 40% of votes and defeated by the other candidate by 298 votes. Find the total number of votes polled.
9. The price of coal is increased by 20%. By what percent a family should decrease its consumption so that expenditure remains same?
10. The price of tea being increased by 20%, a man reduces his consumption by 20%. By how much percent will his expense for tea be decreased?
11. A reduction of 10 % in the price of mangoes enables a man to buy 5 more mangoes for Rs.450. What is the reduced price per mango?
12. Fresh fruits contain 70% of water whereas dry fruit contains 25% of water. How much dry fruit can be obtained from 100 kg of fresh fruits?

PRACTICE QUESTIONS

Numeric

1. $\frac{1}{2}$ is what percent of $\frac{1}{3}$?
2. 125% of 200 + 36% of 350 = x, find x.
3. $3\sqrt{729} + \sqrt{2601} = 60\%$ of x, find x.
4. 125% of 260 + x % of 700 = 500, find x.
5. If 50 % of P = 25% of Q, then P = X% of Q. Find X.
6. 36% of 360 = x, find x.

Arithmetic

1. Sixty-five percent of a number is 21 less than $\frac{4}{5}$ of that number. What is the number?
2. Two numbers are less than the third number by 50% and 54% respectively. By how much percent is the second number less than the first number?
3. Difference of two numbers is 1660 if 7.5% of first number is 12.5% of the other number. Find the two numbers.
4. A number is increased by 20% and then again by 20%. By what percent should increased number be reduced so as to get back the original number?
5. Two students A and B appeared in the same examination. A secured 35% marks in an examination and failed by 15 marks. B secured 165 marks and failed by 25 marks. Find the maximum marks.

6. The mathematics test had 75 problems, in which 10 arithmetic, 30 algebra and 35 geometry problems are there. In that test Sivani answered 70% of arithmetic, 40% of algebra and 60% of geometry problems correctly. She did not pass the test because she got less than 60% of the problems right. How many more questions she would have need to answer correctly to earn a 60% passing grade?
7. A person saves 10% of his income. If his income is increased by 20% and he saves 15% of the new income, then by what percent his actual savings will increase?
8. Vipul decided to donate 5% of his salary. On the day of donation he changed his mind and donated Rs.1687.50 which was 75% of what he had decided earlier. Find his salary?
9. In an election between two candidates, 75% of the voters casted their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of voters enrolled in that election.
10. In an examination, 70% of the candidates passed in English, 80% passed in mathematics, and 10% failed in both subjects. If 144 candidates passed in both, what is the total number of students?
11. Out of 450 students of a school, 325 play football, 175 play cricket and 50 neither play football nor cricket. How many students play both football and cricket?
12. A company has 1500 employees. A given company has 1500 employees. Of those employees, 800 are computer science majors. 25% of those computer science majors are also mathematics majors. That group of computer science/math dual majors makes up one third of the total mathematics majors. How many employees have majors other than computer science and mathematics?
13. The price of wheat falls by 16%. By what percentage a person can increase the consumption of wheat so that his overall budget does not change?
14. If price of sugar is increased by 50%, by what percent a housewife must reduce the consumption of sugar so that her expenditure on sugar is increased by 20% only?
15. When the price of product was decreased by 10% the number sold increased by 30%. What was the increase on total revenue?
16. In a class of 40 students and 5 teachers, each student got sweets that are 15% of the total number of students and each teacher got sweets that are 20% of the total number of students. How many sweets were there?
17. Fresh grapes contain 80% of water while dry grapes contain 10% of water. If the weight of dry grapes is 500gms, then what is its weight when it is fresh?
18. The weight of an alloy of gold and silver is 50 gm. It contains 80% gold. How much gold should be added to the alloy, so that the percentage of gold is increased to 90%?

12. PROFIT AND LOSS

Profit and loss are the terms related to monetary transactions in trade and business. When we sold an article either profit is earned or loss is incurred.

Cost Price (CP)

This is the price at which an article is purchased or manufactured.

Selling Price (SP)

This is the price at which an article is sold.

Overhead Charges

Such charges are the extra expenditures on purchased goods apart from actual cost price. Such charges include freight charges, rent, salary of employees, repairing cost on purchased articles etc.

Profit

When an article is sold at a price more than its cost price, then profit is earned.

$$\checkmark \quad (SP > CP)$$

Loss

When an article is sold at a price lower than its cost price, then loss is incurred.

$$\checkmark \quad (CP > SP)$$

Marked Price (MP)

The price on the label of a product is called marked price or list price.

Discount (D)

The reduction extended on the marked price of a product in effecting its sale is called discount. When there is no discount selling price is equal to marked price.

Types of discount

✓ **Flat discount:**

The discount given at once on the whole amount is called flat discount.

✓ **Successive discount:**

Successive percentage reduction on previously discounted price is called Successive discount.

Tricks to solve the problems

$$\begin{array}{ccc} \text{CP} & \xrightarrow{\text{PROFIT}} & \text{SP} \\ 100\% & \text{or LOSS} & 100\% + P\% \\ & & 100\% - L\% \end{array}$$

$$\begin{array}{ccc} \text{MP} & \xrightarrow{\text{DISCOUNT}} & \text{SP} \\ 100\% & & 100\% - D\% \end{array}$$

DRILL

1. An article costing Rs.400 is sold at 20% profit. Find its selling price.
2. A shopkeeper sold a watch for Rs.540 at the loss of 10%. Find the cost price of watch.
3. A man buys an article for Rs.800 and sells it for Rs.1000. Find profit /loss percent.
4. A person buys a cycle for Rs.450 but because of certain urgency, he sells it for Rs.350. Find his Profit/ Loss percent?
5. A man sells an article at 2% gain. If he sells it at 3% gain he gets Rs.36 more. Find the cost price and selling price of the article.
6. Find the single discount equivalent to the successive discounts of 10%, 5% and 20%?
7. After getting two successive discounts, a shirt with a list price of Rs.150 is available at Rs.105. If the second discount is 12.5%, find the first discount.
8. A dishonest dealer professes to sell his goods at cost price but he uses a weight of 800 grams for 1 kg weight. Find his gain percentage.
9. If the selling price of 50 articles is equal to the cost price of 40 articles then what will be the loss or gain percent?
10. A man sold two watches for Rs.990 each. If he gained 10% on one watch and incurred a loss of 10% on the other, what was the loss or gain percentage in the whole transaction?
11. A man purchased two articles for Rs.2500 each. On selling the first article he gained 10% and on the other he lost 10%. What was his percent profit or loss in the transaction?
12. A shopkeeper buys articles at 20% discount on marked price and sells them at 20% profit.
 - i) How much percent the selling price is less than the marked price?
 - ii) How much percent the marked price is more than the selling price?
 - iii) Find the selling price percentage with respect to the marked price.
 - iv) What is the percentage of marked price to that of selling price?
13. A shopkeeper allows 25% discount on the marked price of his articles and hence gains 25% of the Cost Price. What is the marked price of the article on selling which he gains Rs. 120?

PRACTICE QUESTIONS

1. What is the difference between a discount of 35% of 1800 and two successive discounts of 20% and 15% on the same amount?
2. A dishonest dealer professes to sell his goods at cost price. But he uses a false weight and thus gains $11\frac{1}{9}\%$ for a kg. What is the weight used by him?

3. A dishonest dealer sells his goods at 10% loss on cost price but uses 20% less weight. What is his profit or loss%?
4. A person bought an item at 16% discount on the labeled price. Had he bought it at 20% discount, he would have saved Rs.200. What was the price of the item?
5. A man purchased two articles for total cost of Rs. 9000. He sold the first article at 15% profit and the second at 12% loss. In the bargain, he neither gained nor lost anything. Find the cost price of the first article.
6. Two tables are purchased for the total cost of Rs. 5000. First table is sold at 40% profit and second at 40% loss. If selling price was same for both the tables, what was the cost price of the table that was sold at profit?
7. A man bought a horse and a carriage for Rs 3000. He sold the horse at a gain of 20% and the carriage at a loss of 10%, thereby gaining 2% on the whole. Find the cost of the horse.
8. If the cost price of 12 articles is equal to the selling price of 8 articles, then what is the gain percentage?
9. Six articles were bought for Rs. 5 and sold 5 for Rs. 6. What was the gain percentage?
10. On selling 7 pens for Rs.44, a man gains 10% of the cost price. What should be the selling price of 10 pens if the desired profit is 40%?
11. A person earned a profit by selling an article for Rs.832 which is equal to the loss incurred when the same article is sold for Rs.448. What should be the sale price for making 50% profit?
12. A person sold an article at a loss of 10%. Had he bought it for 20% less and sold it for Rs.55 more, he would have made a profit of 40%. Find the original cost price of the article.
13. By selling an article at $\frac{4}{9}$ of its selling price, a trader incurs 20% loss. What will be the profit percent if the trader sells it at the actual selling price?
14. A merchant marks price of the article 25% above the cost price and allows some discount on the marked price and makes a profit of 12.5%. Find the rate of discount.
15. If 10% discount is allowed on the marked price then profit is 20%. If the discount is increased to 20%, then what will be the profit?

13. SIMPLE INTEREST

Principal

Money borrowed or lent out for a certain period is called the principal amount or sum.

Simple Interest

If the interest on a sum of money borrowed for a certain time period is reckoned uniformly, then it is called as Simple Interest.

$$S.I = \frac{P \times N \times R}{100}$$

Amount (A) = Principal (P) + Interest (I)

DRILL

1. Find the simple interest of Rs.7300 for 3 years at 10% per annum.
2. Find the simple amount on Rs.200 for 5 year at 6% per annum.
3. If simple interest on certain sum at 4% per annum for 4 years is Rs.80, then find the sum.
4. A sum of Rs.800 becomes Rs.1152 in 4 years. What is the rate of interest per annum?
5. Pinky invested an amount of Rs.2500 at the rate of 6% per annum. After how many years will she get a simple interest of Rs.600?
6. At a certain rate of simple interest Rs.800 becomes Rs.956 in 3 years. If the rate of interest is increased by 4%, what amount will Rs.800 become in 3 years?
7. A sum is lent for 6 yrs at 37% and 33% simple interest respectively and the difference of interest is Rs.54. Find the sum.
8. A person lent a certain sum at 8% per annum simple interest and in 7 years the interest amounted to Rs.2244 less than the principle amount. What is the amount that the person had lent?
9. In what rate of interest will a sum of money double itself at 5 years?
10. A sum of money triples at 4% interest and it will be 6 times in what rate of interest?
11. A sum of money doubles in 5 years and it becomes 4 times in how many years?
12. In simple interest how many years will a sum of money become 4 times at 12% per annum?

PRACTICE QUESTIONS

1. In what time Rs.72 become Rs.81 at $6\frac{1}{4}\%$ per annum at simple interest?
2. A sum of money at simple interest amounts to Rs.815 in 3 years and to Rs.854 in 4 years. Find the sum.
3. A sum of money lent out at simple interest amounts to Rs.720 after 2 years and to Rs.1020 after a further period of 5 years. Find the sum.
4. The simple interest on a certain sum of money at the rate of 5% per annum for 8 years is Rs.840. At what rate of interest the same amount of interest can be received on the same sum after 5 years?
5. If a sum of money at a certain rate of simple interest doubles in 5 years and at a different rate of interest becomes 3 times in 12 years. Which is the better rate of interest?
6. In how many years will a sum of money double itself at 12% per annum Simple interest?
7. The simple interest accrued on a certain sum is $\frac{1}{4}$ th of the sum .Find the rate of interest when the time is 16 years.
8. The simple interest accrued on a certain sum is $\frac{1}{4}$ th of the sum .Find the sum and Simple interest when the time is 16 years.
9. The simple interest accrued on a certain sum is $\frac{1}{4}$ th of the sum and the number of years is equal to the rate of percentage per annum. What is the rate of interest per annum?
10. The simple interest obtained on a certain sum is $\frac{16}{25}$ th of the sum .Find the rate of interest and time if both are numerically equal.
11. A sum of Rs.600 is lent out in three parts in such a way that the interest on first part at 3% for 4 yr, second part at 4% for 5 yr and third part at 5% for 3 yr. If interest received on each part is equal, then find the amount interested at 5%.
12. If the simple interest on a certain sum for 15 months at 7.5% per annum exceeds the simple interest on the same sum for 8 months at 12.5% per annum by Rs.32.50, then what will be the sum?

14. COMPOUND INTEREST

Compound interest can be defined as interest applied on the prior interest in addition to that on the principal i.e., Compound interest is referred to as interest on interest. It grows exponentially.

$$C.I = P \left(1 + \frac{R}{100} \right)^N - P$$

- ✓ When interest is compounded annually

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^N$$

- ✓ When interest is compounded Half- yearly, then $R = \frac{R}{2}$ and $N = 2N$

$$\text{Amount} = P \left(1 + \frac{R}{2 \times 100} \right)^{2N}$$

- ✓ When interest is compounded quarterly, then $R = \frac{R}{4}$ and $N = 4N$

$$\text{Amount} = P \left(1 + \frac{R}{4 \times 100} \right)^{4N}$$

- ✓ If rates of interest are $R_1\%$, $R_2\%$ and $R_3\%$ for 1st, 2nd and 3rd respectively, then

$$\text{Amount} = P \left(1 + \frac{R_1}{100} \right)^N \left(1 + \frac{R_2}{100} \right)^N \left(1 + \frac{R_3}{100} \right)^N$$

DRILL

1. Find compound interest on Rs.12000 for 2 years the rate of interest being 4% per annum?
2. What will be compound amount on a sum of Rs.25000 after 3 years at the rate of 20% per annum?
3. If Rs.8000 amounts to Rs. 9261 in 3 years at compound interest, then find the rate of interest per annum?
4. If Rs.8000 amounts to Rs. 8820 in 2 years at compound interest, then find the rate of interest per annum?
5. The population of a town increase by 10 % annually. If the present population is 2000, then after how many years the population will be 2662?
6. In how many years will Rs 4000 amount to Rs 4630.50 at 5% per annum compounded annually?
7. Compute the compound interest on Rs.12,000 for 1½ years at 10% p.a. when compounded half-yearly.
8. Find the compound interest on a sum of Rs.4000 at the rate of 20% per annum for 9 months, compounded quarterly.

9. A certain sum of money amounts to Rs.640 after 2 years and Rs.720 after 3 years . How much will it amount in 4 years?
10. Present population of a town is 12000. Increasing at a certain rate the population becomes 27000 after 6 years .What will be the population after 3 years from now?
11. A sum of money placed at compound interest doubles itself in 4 years. In how many years will it amount to 8 times?
12. A sum of money placed at compound interest triples itself in 10 years. In how many years will it amount to 9 times?
13. The difference between simple and compound interest (compounded annually) on a sum of money for 2 years at 10% per annum is Rs. 255. Find the sum.
14. The difference between simple interest and compound interest for 3 years at the rate of 20% per annum is Rs.144. What is the principle amount?
15. If simple interest for 3 years at 10% is Rs.900, then what will be the compound interest?

PRACTICE QUESTIONS

1. Find the compound interest on Rs.25000 at 20% per annum for $2\frac{1}{2}$ years if interest is compounded annually?
2. Gajendran invested an amount of Rs.8000 in a fixed deposit scheme for 2 years at compound interest rate 5 p.c.p.a. How much amount will Gajendran get on maturity of the fixed deposit?
3. If compound interest on a certain sum for 2 years at 10% per annum is Rs. 315.What is the sum lent?
4. What is the principle if the compound amount, compounded annually, at the rate of 10% per annum for 3 years in Rs.14641?
5. At what percent annual compound interest rate, a certain sum amounts to its 27 times in 3 years?
6. A sum of money at compound interest amounts to Rs.2916 in 2 years and Rs.3149.28 in 3 years. Find the principle.
7. Divide Rs.2602 between A and B at that the rate of interest 4% where amount of A after 7 years is equal to the amount of B after 9 years?
8. If simple interest and compound interest on a certain sum for 2 years are Rs.400 and Rs.440 respectively .What is the principle?
9. If compound interest on a certain sum for 3 year at 10% is Rs.662, then find simple interest on that sum for 3 years?

15. ALLIGATION OR MIXTURE

Mixture

Combination of two or more quantities is called as mixture.

Alligation

Alligation is the rule that enables us to find the ratio in which two or more ingredients/ things have been mixed.

Mean price

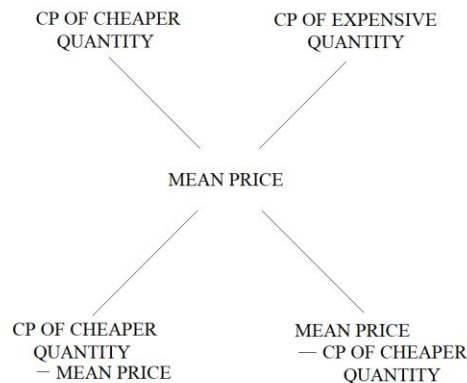
The cost price of a unit quantity of the mixture is called the mean price. It will always be higher than cost price of cheaper quantity and lower than cost price of expensive quantity.

Rule of alligation

The basic formula which is used to find the ratio in which the ingredients are mixed is

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of expensive}} = \frac{\text{Cost price of expensive} - \text{Mean price}}{\text{Mean price} - \text{Cost price of cheaper}}$$

It is called as the rule of alligation and can also be represented as



- If a container contains x units of liquid from which y units are taken out and replaced with b unit of another liquid. After n operations, the final quantity of the original liquid in the container is given as $\left[a \left(1 - \frac{b}{a} \right)^n \right]$ units.

DRILL

1. In what proportion must wheat at Rs.6.20 per kg mixed with wheat at Rs. 7.20 per kg, so that the mixture is worth Rs. 6.50 per kg?
2. The mean weight of 150 students in a certain class is 60 kg. The mean weight of the boys from the class is 70 kg, while that of girls is 55 kg. What is the number of girls in the class?
3. In an examination, a student gets 3 marks for every correct answer and loses 1 mark for every wrong answer. If he scores 100 marks in a paper of 100 question, then how many of his answers are correct assuming that he attempts all the questions?

4. A merchant has 50 kg of pulse. He sells one part at a profit of 10% and other at 5% loss. Over all he has a gain of 7%. Find quantity of pulses sold at 10% profit and 5% loss.
5. In what ratio must a grocer mix two varieties of tea worth Rs.60 per kg and Rs.65 per kg, so that by selling the mixture at Rs.68.20 per kg he may gain 10%?
6. A person has a chemical of Rs.25 per liters. In what ratio should water be mixed in that chemical so that after selling the mixture at Rs.20 per liters he may get a profit of 25%?
7. A man invested Rs.3500 into two parts at simple interest, one at 4% and other at 6% per annum. If his yearly income from investment was Rs.170, how much did he invest at 6%?
8. Two containers have milk and water in the ratio 2:1 and 3:1 respectively. If both the containers are emptied into a bigger container, then find the ratio of milk to water in the bigger container?
9. The milk and water in two vessels A and B are in the ratio 4:3 and 2:3 respectively. In what ratio liquids in both the vessels should be mixed to obtain a new mixture in vessel C containing half milk and half water?
10. In what ratio must water be mixed with milk to $16\frac{2}{3}\%$ on selling the mixture at cost price?
11. A dishonest milkman professes to sell his milk at cost price but he mixes it with water and thereby gains 25%. Find the percentage of water in the mixture.
12. If 50 liters of milk solution has 40% of milk in it, then how much milk should be added to make it 60% in the solution?
13. In a container, milk and water are present in the ratio 7:5. If 15 liters of water is added to the mixture, the ratio of milk and water becomes 7:8. Find the quantity of water in the new mixture.
14. A mixture of 40 liters contains milk and water in the ratio 4:1. How many liters of mixture must be replaced with water so that the ratio is 1:1?
15. A mixture contains milk and water in the ratio 5:3. If 16 liters of mixture is replaced by 16 liters of water, then the ratio of milk and water becomes 3:5. How much milk was there in the mixture initially?
16. A jar full of sugar solution contains 40% water. A part of this sugar solution is replaced by another which contains 19% water and now the percentage of water is found to be 26%. Find the percentage of sugar solution replaced.
17. A solution contains 10% salt. One fourth of this solution is replaced by a second solution, resulting in a solution that contains 16% salt. Find the percentage of salt in the second solution.

18. A vessel contains 100 liters of milk. From this vessel 10 liters of milk is taken out and replaced by water. The process is repeated further two times. How much milk is now contained by the vessel?
19. A vessel contains 40 liters of milk. From this certain liters of milk is taken out and replaced by water. The process is repeated further two times and the remaining milk is 29.16 liters. How much milk is taken out and replaced every time?
20. 8 liters is drawn from a cask full of acid and then filled with water. This operation is performed three more times. The ratio of the quantity of acid now left in cask to that of the water is 16:65. How much acid did the cask hold originally?

PRACTICE QUESTIONS

1. The cost of Type 1 rice is Rs. 15 per kg and Type 2 rice is Rs. 20 per kg. If both Type 1 and Type 2 are mixed in the ratio of 2 : 3, then find the price per kg of the mixture.
2. A merchant mixes two varieties of chemicals containing 25% potassium and 13% potassium. The resultant mixture contains 17% potassium. Find the quantity of the second variety, if 8 kg of the first mixture is taken?
3. A merchant has 200 kg of rice. He sells one part at 36% profit and the rest at 16% profit. He gains 28% on the whole. Find the quantity sold at 16%.
4. A person had Rs.8400. He lent a part of it at 4% and the remaining at $3\frac{1}{3}\%$ SI. His total annual income was Rs.294. Find the sum lent at 4%.
5. In a zoo, there are rabbits and pigeons. If heads are counted, there are 340 heads and if legs are counted there are 1060 legs. How many pigeons are there?
6. In a 20 liters mixture of milk and water, water is only 25%. How many liters of water should be added to the mixture to increase the percentage of water to 80%?
7. A vessel is filled with liquid, 3 parts of which are water and 5 parts are syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?
8. A can contains a mixture of two liquids A and B in the ratio 7:5. When 9 liters of mixture has drawn off and the can is filled with B, the ratio of A and B becomes 7:9. How many liters of liquid A was present in the can initially?
9. One type of liquid contains 25% milk and another contains 30% of milk. A can is filled with 6 part of 1st and 4 part of 2nd liquid. Find the percentage of milk in the new mixture
10. 5 liters of milk is taken out from a container full of milk and filled with water. Again 5 liters of mixture is drawn and the container is filled with water. If the ratio of milk and water now left in the container is 16:9, then find the capacity of the container.

16. AVERAGES

The average is defined as the mean value which is equal to the sum of the number of a given set of values divided by the total number of values present in the set.

$$\text{Average} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

Important formulae related to Average of numbers

- ✓ Average of first n natural numbers = $\frac{n+1}{2}$
- ✓ Average of first n even numbers = $(n + 1)$
- ✓ Average of first n odd numbers = n
- ✓ Average of consecutive numbers = $\frac{\text{First number} + \text{Last number}}{2}$
- ✓ Average of 1 to n odd numbers = $\frac{\text{Last odd number} + 1}{2}$
- ✓ Average of 1 to n even numbers = $\frac{\text{Last even number} + 2}{2}$
- ✓ Average of squares of first n natural numbers = $\frac{(n+1)(2n+1)}{6}$
- ✓ Average of the cubes of first n natural numbers = $\frac{n(n+1)^2}{4}$
- ✓ Average of n multiples of any number = $\frac{\text{Number} \times (n+1)^2}{2}$

DRILL

1. Find the average of first 41 natural numbers.
2. Find the average of 2, 4, 6, 8, 10, 12 and 14.
3. Average of 5 consecutive even numbers A, B, C, D and E is 22. Find product of A & E.
4. The average of four consecutive even numbers A, B, C and D is the smallest number among four consecutive odd numbers E, F, G and H, where the average of G and H is 22. Find the value of A?
5. The average of all three numbers is 180. Among three numbers the first is thrice the second number and the second number is twice the third number. Find the first number.
6. The sum of five numbers is 555. The average of first two numbers is 75 and the third number is 115. What is the average of last two numbers?

7. The average age of a family of 4 members is 20 years old. If the present age of the youngest member is 4 years old, then what was the average age of the family at the time of birth of the youngest member?
8. The mean weight of 34 students of a school is 42 kg. If the weight of the teacher be included, the mean rises by 400gms. Find the weight of the teacher (in kg).
9. The average age of 14 girls and their teacher's age is 15 years. If the teacher's age is excluded, then the average age is reduced by one year. What is the teacher's age?
10. In a class there are 30 students. Their men weight is increased by 200g, when one student whose weight is 20kg is replaced by a new student. Find the weight of the new student.
11. The average marks secured by 9 students were 65. Later on it was found that the mark of one student was wrongly read as 76 instead of 67. What was the correct average?
12. In a certain examination, the average mark of an examinee is 64 per paper. If he had obtained 18 more marks for his mathematics paper and 4 more marks for his English paper, his average per paper would have been 66. How many papers were there in the examination?
13. The average of runs of a cricket player for 10 innings was 32. How many runs must he make in his next innings so as to increase his average of runs by 4?

PRACTICE QUESTIONS

1. The average of 5 consecutive numbers A B C D and E is 105. What is the product of B and E?
2. The sum of 5 consecutive odd numbers A, B, C, D and E is 105. Find product of B & E.
3. The average of nine numbers is 50. The average of the first five numbers is 54 and that of the last three numbers is 52. Find the sixth number.
4. The average of 4 numbers is 67.5. The highest and the smallest numbers are 108 and 23 respectively. If the difference between other two numbers is 31, then find the bigger number of the two.
5. The average weight of A, B and C is 45 kg. If the average weight of A and B is 40 kg and that of B and C is 43 kg, then find the weight of B.
6. The average weight of the students of a class is 40 kg. 5 new students with the average weight of 46 kg have joined the class, the average weight of the class increased by 2 kg. Find the number of students in the class originally?
7. A cricketer scored some runs in his continuous 9 innings. He scored 100 runs in his 10th innings and increases his average by 8 runs. What was the average of his runs at the end of 10th innings?

17. TIME AND WORK

- ✓ If a person can do a piece of work in n days, then the person's 1 day work = $1/n$.
- ✓ If a person's 1 day work = $1/n$, then the person can finish the work in n days.

Important relations

- ✓ Work done and Number of person are directly proportional.
- ✓ Work done and Time are directly proportional.
- ✓ Time and Number of person are indirectly proportional.
- ✓ Time and Efficiency are indirectly proportional.
- ✓ Wages and Number of person are directly proportional.

Therefore,

$$\frac{M_1 D_1 H_1 E_1}{W_1} = \frac{M_2 D_2 H_2 E_2}{W_2}$$

Where, M = Number of workers

D = Number of days

H = Number of working hours in a day

E = Efficiency of workers

W = Units of work / wages

Efficiency

- ✓ If A is twice as good a workmen as B, then
 - Ratio of work done by A and B in the same time = 2:1
 - Ratio of time taken by A and B in doing the same work = 1:2
- ✓ Wages are given in proportion to the work done and in indirect (or inverse) proportion to the time taken by the individual.

DRILL

1. A finishes the work in 10 days. B finishes the same work in 15 days. In how many days they working together will finish the work?
2. A, B and C can complete a piece of work in 24, 8, 12 days respectively. Working together, they will complete the same work in how many days?
3. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work?
4. A and B together can complete a piece of work in 18 days, B and C in 24 days, A and C in 36 days. In how many days will A, B and C finish it by working together and separately?

5. A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?
6. A & B undertook to do a piece of work for Rs.4500. A alone could do it in 8 days and B alone in 12 days. With the assistance of C, they finished the work in 4 days. Find C's share.
7. A can do a work in 60 days. He works for 15 days and later B alone completes the remaining work in 30 days. In what time will they together complete the work?
8. A and B can do a piece of work in 28 and 35 days respectively. They began to work together but A left after sometime and B completed the remaining work in 17 days. After how many days did A leave?
9. A completes 80% of a work in 20 days. Then B also joins and A and B together finish the remaining work in 3 days. How long does it need for B if he alone completes the work?
10. If A and B working separately can do a piece of work in 10 and 12 days respectively, and if they work for a day alternately, how many days will the work be completed with A beginning the work?
11. 10 men, working 6 hours a day can complete a work in 18 days. How many hours a day must 15 men work to complete the work in 12 days?
12. If 20 men can build a wall 56 meters long in 6 days, what length of similar wall can be built by 35 men in 3 days?
13. If the wages of 6 men for 15 days is Rs. 2100, then find the wages of 9 men for 12 days.
14. 8 men can do a piece of work in 24 days. After 6 days of work 4 more men were engaged to finish the work. In how many days would the remaining work be completed?
15. Sixteen men and twelve women can complete a work in 8 days. If 20 men can complete the same work in 16 days, in how many days 16 women can complete the same piece of work?
16. 5 men can do a piece of work in 6 days while 10 women can do it in 5 days. In how many days can 5 women and 3 men do it?
17. 6 men or 8 women can reap a field in 86 days. How many days will 14 men and 10 women take to reap it?
18. If 2 men and 3 boys can do a piece of work in 10 days, while 3 men and 2 boys can do the same work in 8 days, then in how many days can 2 men and 1 boy do the same work?

PRACTICE QUESTIONS

1. A can finish a work in 18 days and B can do same work in half the time taken by A. If they work together, what part of same work they can finish in a day.
2. Suman can do a work in 3 days. Sumati can do the same work in 2 days. Both of them finish the work together and get Rs. 150. What is the share of Suman?
3. A and B can do a piece of work in 20 days & 15 days respectively. A started the work alone and then left after 4 days. If B completed the remaining work, then how long did the work last?
4. A can do a piece of work in 48 days and B can do the same piece of work in 32 days.. Both of them worked together for 15 days and then B left the work. In how many days will A finish the work?
5. A & B working separately can do a piece of work in 9 & 12 days respectively. If they work for a day alternatively, beginning with A, in how many days the work will be completed?
6. 36 workers can finish a piece of work in 14 days. If the work is to be completed in 8 days, how many extra workers are required?
7. A garrison of 850 men has enough provisions for 26 weeks. If at the end of 6 weeks, they are reinforced by 150 men, how long would the provisions then last?
8. A contractor undertakes to make a road in 40 days and employs 25 men. After 24 days he finds only one third is made. How many extra men should he employ so that he is able to complete the work 4 days earlier?
9. A contractor agreeing to finish a work in 150 days employed 75 men each working 8 hours daily. After 90 days, only $\frac{2}{7}$ of the work was completed. How many extra men should employ so that the work will be completed on time if each working 10 hrs daily?
10. B is 50% less efficient than A, and C is 25% more efficient than B. B alone can complete the work in 60 days. If A, B, C together started the work, A left 1 day before and B left 6 days before completion of work, then find number of days for which A worked?
11. M can do a work in 10 hours and N can do the same work in 15 hours. If they start working together but stop working after four hours, find the fraction of work that is left?
12. 12 men and 15 women can do a piece of work in 10 days and 21 women and 24 children can finish the same work in 15 days. In how many days 1 man, 3 women and 2 children together finish the same work, at their double efficiency?
13. If 9 men or 15 women can do a piece of work in 18 days working 9 hours a day. How many days will it take to complete a work twice as large with 6 men and 8 women working together 6 hours a day?

18. PIPES AND CISTERNS

DRILL

1. Two pipes A and B can fill a tank in 24 hours and 30 hours respectively. If both the pipes are opened simultaneously in the empty tank, how much time will be taken by them to fill it?
2. A can fill the tank in 4 hours and B can empty it in 9 hours. If both are opened simultaneously, how much time will it take to fill the tank?
3. A pipe can fill a tank in 20 hrs. Due to a leak in the bottom, it is filled in 25 hrs. If the tank is full, how much time will it take to empty the tank?
4. A full tank gets emptied in 6 min due to presence of orifice in it. On opening a tap which can fill the tank at the rate of 8 lt/min, the tank gets emptied in 10 min. Find the capacity of the tank
5. If tap A can fill a tank 3 times faster than tap B and takes 28 min less than tap B to fill the tank . If both the taps are opened simultaneously then find the time taken to fill the tank.
6. Two pipes A and B can fill a tank in 36 min and 45 min respectively. Another pipe C can empty the tank in 30 min. First A and B are opened. After 7 minutes, C is also opened. In how much time, the tank will be filled?
7. Two pipes A and B can fill a tank in 18 and 24 min respectively. If both the pipes are opened together, after how much time pipe B should be closed so that the tank is filled in 12 min?
8. A tank has three pipes. The first pipe can fill $\frac{1}{2}$ part of the tank in 1 hour and the second pipe can fill $\frac{1}{3}$ part in 1 hour. The third pipe is for making the tank empty. When all the three pipes are open, $\frac{7}{12}$ part of the tank is filled in 1 hour. How long will third pipe take to empty the tank?

PRACTICE QUESTIONS

1. An outlet pipe can empty a cistern in 5 hr. In what time will the pipe empty two-fifth part of the cistern?
2. Three taps A, B, and C can fill a tank in 18, 24, and 36 hours respectively. If all three pipes are opened simultaneously, in how much time will the tank get half filled?
3. Inlet A is four times faster than inlet B to fill a tank. If A alone can fill it in 15 min, how long will it take if both the pipes are opened together?

4. A cistern is normally filled in 8 hours but takes another 2 hours longer to fill because of a leak in its bottom. If the cistern is full, how long will the leak take to empty the tank?
5. A filling tap can fill a tank in 10 hour. Two equivalent filling tap and one outlet tap are open simultaneously then tank is filled in $7\frac{1}{2}$ hours. In how much time outlet tap can empty the tank.
6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom, 32 minutes extra are taken for the cistern to be filled up. When the cistern is full in what time will the leak empty it?
7. Pipes A and B can fill a cistern in 20 min and 30 min and C can empty it in 15 min. If the three are kept open successively for 1 min each in the order, then how soon the cistern is filled?
8. A pipe can fill a cistern in 12 min and another pipe can fill it in 15 min, but a third pipe can empty it in 6 min. The first two pipes are kept open for 5 min in the beginning and then the third pipe is also opened. In what time is the cistern emptied?
9. Two pipes A and B can fill a tank in 24 and 32 min respectively. If both the pipes are opened together, after how much time pipe B should be closed so that the tank is full in 18 min?
10. A cistern has three pipes A, B and C. The pipes A and B can fill it in 3 and 4 hours respectively and C can empty it in 1 hours. At what time will the cistern be emptied, if the pipes are opened at 3, 4 and 5 pm respectively?
11. Pipe A can fill a tank in 15 min and pipe B can drain 40 lt/min. If both the pipes are opened together, the cistern is filled in 45 min. Find the capacity of the cistern.
12. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely?
13. There is an empty reservoir whose capacity is 30 litres. There is an inlet pipe which fills at 5 L/min and there is an outlet pipe which empties at 4 L/min. Both the pipes function alternately for 1 minute. Assuming that the inlet pipe is the first one to function, how much time will it take for the reservoir to be filled up to its capacity?

19. TIME AND DISTANCE

Speed

The rate at which an object travels to cover a certain distance is called speed of that object.

Time

The duration in hours, minutes or seconds spent to cover a certain distance is called the time.

Distance

The length of the path travelled by any object or a person between two places is known as distance.

Relation between Speed, Time and Distance

Speed is the distance covered by an object in unit time. It is calculated by an object in unit time. It is calculated by dividing the distance travelled by the time taken.

- ✓ **Speed = Distance / Time**
- ✓ **Time = Distance / Speed**
- ✓ **Distance = Speed × Time**

Unit Conversion

$$1 \text{ km} = 1000 \text{ mtr}$$

$$1 \text{ hr} = 3600 \text{ sec}$$

- ✓ **To convert km/hr into m/sec**

$$\begin{aligned} 1 \text{ km/hr} &= \frac{1000}{3600} \text{ m/s} \\ &= \frac{5}{18} \text{ m/s} \end{aligned}$$

- ✓ **To convert m/sec to km/hr**

$$\begin{aligned} 1 \text{ m/s} &= \frac{3600}{1000} \text{ km/hr} \\ &= \frac{18}{5} \text{ km/hr} \end{aligned}$$

Proportionality Constants

- ✓ When the distance covered is constant, the speed is inversely proportional to time.

$$\text{i.e., } S \propto \frac{1}{T}$$

- ✓ When the Time is constant, the distance covered is directly proportional to speed.

$$\text{i.e., } D \propto S$$

- ✓ When the speed is constant, the distance covered is directly proportional to time.

$$\text{i.e., } D \propto T$$

Average Speed

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

- ✓ **When the distance travelled is same at different speeds**

If a man covers a distance D with speed S_1 in time T_1 and again covers the same distance D at speed S_2 in the same time T_2 , then

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{D + D}{T_1 + T_2} \\ &= \frac{2D}{\frac{D}{S_1} + \frac{D}{S_2}} \\ &= \frac{2 S_1 S_2}{S_1 + S_2} \end{aligned}$$

- ✓ **When the time taken to travel is same for different distance**

If a man covers a distance D_1 with speed S_1 in time T and covers another distance D_2 with speed S_2 in the same time T , then

$$\begin{aligned} \text{Average speed} &= \frac{\text{Total distance travelled}}{\text{Total time taken}} \\ &= \frac{D_1 + D_2}{T + T} \\ &= \frac{S_1 \times T + S_2 \times T}{T + T} \\ &= \frac{S_1 + S_2}{2} \end{aligned}$$

Relative Speed

- ✓ When A and B are moving in opposite direction at speed of S_1 and S_2 respectively, then

$$\text{Relative speed} = S_1 + S_2$$

- ✓ When A and B are moving in same direction at the speed of S_1 and S_2 respectively, then

$$\text{Relative speed} = S_1 - S_2$$

DRILL

1. If a car covers 125 km in 5 hr, then find the speed of the car.
2. A man runs 100 m at the speed of 30 km/hr. In what time does he cover the distance?
3. The speed of the bus is 72 km/hr. What is the distance covered by the bus in 5 seconds?
4. Convert 90 km/hr into m/s.
5. If speed of $3\frac{1}{3}$ m/s is converted to km/hr, then what would be the result?
6. A man covered a distance of 12 km in 90 min by cycle. How much distance will he cover in 3 hr, if he rides the cycle at a uniform speed?
7. A person covers a distance of 12 km while walking at a speed of 4 km/hr. How much distance he would cover in same time, if he walks at a speed of 6 km/hr?
8. A person covered a certain distance by bus at the rate of 40 km/hr and walked back into the initial point at the rate of 6 km/hr. The whole journey took 13 hrs 48 min. What distance did he walk?
9. A person covers a distance of 20 km by bus in 35 min. After deboarding the bus, he takes rest for 15 min and covers another 10 km by a taxi in 10 min. Find his average speed for the whole journey.
10. Shantanu covers a certain distance by car driving at 35 km/hr and he returns back to the starting point riding on a scooter with a speed of 15 km/hr. Find the average speed of the whole journey.
11. Two persons are moving in the direction opposite to each other. Speeds of both the persons are 5 km/hr and 3 km/hr respectively. Find the relative speed of the two persons in respect to each other.
12. Two trains are running in the same direction. The speeds of two trains are 5 km/hr and 15 km/hr respectively. What will be the relative speed of the second train with respect to that of first one?
13. A student rides on a bicycle at 6 km/hr and reaches his school 5 min late. The next day he increased his speed to 10 km/hr and reaches school 5 min early. How far is the school from the house?
14. Walking $\frac{6}{7}$ th of his usual speed, a man is 12 min late. Find the usual time taken by him to cover the distance.

15. A policeman sees a chain snatcher at a distance of 50 m. He starts chasing the thief who is running with a speed of 2 m/s while the speed of the policeman is 4 m/s. Find the distance covered by the thief when he is caught by the policeman.
16. A policeman sees a thief who is at a distance of 600 m. He chases him at a speed of 10 km/hr and catches him in 12 min. Find the speed of the thief in km/hr.

PRACTISE QUESTIONS

1. If a car covers 300 km in 15 hr, then find the speed of the car.
2. A train covers a distance of 200 km with a speed of 10 km/hr. What is the time taken by the train to cover this distance?
3. A bike covers a bridge with a speed of 108 km/hr. What will be the length of the bridge, if the bike takes 8 hr to cross the bridge?
4. Convert 72 km/hr into m/s.
5. Convert 35 m/s into km/hr.
6. A person covers 21 km in 3 hr. What distance will he cover in 5 hr?
7. A person covers a certain distance in 6 hours if travels at 40 km/hr. At what speed he must travel if he has to cover the same distance in 4 hours?
8. If a person covers 40 km at a speed of 10 km/hr by a cycle, 25 km at 5 km/hr on foot and another 100 km at 50 km/hr by bus, then find his average speed of the whole journey.
9. A boy walking at a speed of 20 km/hr reaches his school 30 min late. Next time he increases his speed by 4 km/hr but still he is late by 10 min. Find the distance of the school from his house.
10. A man increases his speed to $\frac{7}{5}$ times of his original speed and reaches his office 20 min before the fixed time. Find the usual time taken by him to reach the office.
11. Excluding stoppages, the speed of the bus is 54 km/hr and including stoppages, its speed is 45 km/hr. For how many minutes does the bus stop per hour?
12. A thief steals a car at 2.30 pm and drives at 60 km/hr. The thief is discovered at 3 pm and the owner sets off in another car at 75 km/hr. When will he overtake the thief?
13. A man takes 5 hrs 45 min in walking to a certain place and riding back. He would have gained 2 hrs by riding both ways. What is the time he would take to walk both ways?
14. A bus can travel 50% faster than a car. Both start from point A at the same time and reach point B 75 kilometers away from A at the same time. On the way, however, the bus lost about 12.5 minutes while stopping at the bus stop. Find the speed of the car.

20. RACES

The competitions of running, driving cars, riding on horses or bicycles, rowing boats, etc., are generally called **races**. The places like grounds, roads, rivers, lakes, etc., where races take place are called the **race course**. In any race, the number of participants can be 2 or more.

The point from where the race starts is known as the **starting point**. The point at which the race ends is called the **goal** or **winning post**.

If in a race all participants reach the goal at the same time, the race is called **dead heat**. Therefore, dead heat is the type of race which ends in a draw, i.e., no one is winner or loser.

Linear Track

If A and B are two contestants in a race at linear track, the following scenarios may occur

- ✓ While starting the race, if A is at the starting point and B is ahead of A by 20 meters, we say that “A gives B a start of 20 meters”.
In this case, to cover a race of 100 meters, A will have to cover 100 meters while B will have to cover only $(100 - 20) = 80$ meters.
- ✓ “A beats B by 20 meters” means A won the race when B was beyond by 14 meters.
In this case, while A runs 100 m, B is at $(100 - 20) = 80^{\text{th}}$ meter.
- ✓ “A can give B 20 m” means while A runs 100 m, B runs $(100 - 20) = 80$ m.
- ✓ “A game of 100” means that the person among the participants who scores 100 points first is the winner'.
- ✓ If A scores 200 points while B scores only 160 points, we can say that “A can give B 40 points”.

Circular Race Track

Racing on a circular track means running on a circular path such that the starting point and the finishing point coincide with each other. It is different from linear tracks in the way that a race on a circular track can have a number of laps and the participants go around the same track again and again.

If the race is conducted in a circular track, the following scenarios may occur.

- ✓ They can meet at starting point while racing.
- ✓ They can meet at anywhere on the circular track for the first time while racing (not necessarily at the starting point).
- ✓ There may be many number of meeting points while moving in same direction
- ✓ There may be many number of meeting points while moving in opposite direction.

- i. If two runners start at the same time and from the same point and run in the same direction, when they meet for the first time, the faster runner would have run one round more than the slower runner. Here Relative Speed is $S_1 - S_2$.

$$\text{Time taken for their first meeting} = \frac{\text{Length of the track}}{S_1 - S_2}$$

- ii. If two runners start at the same time and from the same point and run in the opposite direction, they meet on the first lap for the first time. Here Relative Speed is $S_1 + S_2$.

$$\text{Time taken for their first meeting} = \frac{\text{Length of the track}}{S_1 + S_2}$$

- iii. When three runners start from the same point at the same time in same direction, the time taken for their first meeting is the LCM of the time taken by the fastest runner to get a lead of one complete round over each of the other two.

$$\text{Time taken for their first meeting} = \text{LCM} \left(\frac{\text{Length of the track}}{S_1 - S_2}, \frac{\text{Length of the track}}{S_2 - S_3} \right)$$

- iv. If two or more runners start at the same time and from the same point, the time taken for them to meet **at the starting point** for the first time is the LCM of the durations they take to complete one round. The directions of racing is immaterial (makes no difference).

$$\begin{aligned} \text{Time taken for their first meeting at the starting point} \\ = \text{LCM (Time taken by each to complete one round)} \end{aligned}$$

- v. When moving in circular track, the runners can meet several times. To find the number of meeting points, we need to find the time taken for their first meeting at starting point and time taken for their first meeting at any point.

$$\text{Number of meeting points} = \frac{\text{Time taken for their first meeting at the starting point}}{\text{Time taken for their first meeting at any point}}$$

DRILL

- In a kilometer race, A beats B by 40 meter or 8 seconds. Find A's time over the course.
 - 1 min 12 sec
 - 2 min 12 sec
 - 3 min. 12 sec
 - 4 min 12 sec
- A can run 1 km in 3 min 10 sec and B can cover the same distance in 3 min 20 sec. By what distance can A beat B?
 - 80 m
 - 70 m
 - 60 m
 - 50 m
- In 100m race, A runs at 6 kmph. If A gives B a start of 4m and still beats him by 12 seconds, what is the speed of B?
 - 1.8 km/hr
 - 2.8 km/hr
 - 3.8 km/hr
 - 4.8 km/hr
- A, B, C are the three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64 m, how many meters start can B give C?
 - 25 m
 - 20 m
 - 35 m
 - 45 m

5. In a race of 600m, A can beat B by 60m and in a race of 500m, B can beat C by 50m. By how many meters will A beat C in a race of 400m?
 a) 46 m b) 56 m c) 66 m d) 76 m
6. In a 500 m race, the ratio of the speeds of A and B is 3:4. A has a start of 140m. By what distance A wins the race.
 a) 10m b) 20m c) 40m d) 60m
7. At a game billiards, A can give B 10 points in 60 and he can give C 15 points in 60. How many points can B give C in a game of 90?
 a) 9 points b) 8 points c) 7 points d) 6 points
8. A and B take part in a 100m race. A runs at 5km/hr. A gives B a start of 8m and still beats him by 8 sec. Find the speed of B.
 a) 5.15 km/hr b) 4.15 km/hr c) 4.25 km/hr d) 4.14 km/hr
9. In a circular race of 4500 m, A and B start from the same point and at the same time with speeds of 36 kmph and 54 kmph. Find when will they meet again for the first time on the track when they are running in the same direction and opposite direction?
 a) 900sec, 50sec b) 800sec, 60sec c) 800 sec, 50sec d) None of these
10. Three bodies A, B and C start moving around a circular track of length 60m from the same point simultaneously in the same direction at speeds of 3 m/s, 5 m/s and 9 m/s respectively. When will they meet for the first time after they started moving?
 a) 30 seconds b) 60 seconds c) 15 seconds d) 10 seconds
11. Three bodies A, B and C start moving around a circular track of length 60m from the same point simultaneously in the same direction at speeds of 3 m/s, 5 m/s and 9 m/s respectively. When will they meet for the first time at the starting point after they started moving?
 a) 30 seconds b) 60 seconds c) 15 seconds d) 10 seconds
12. In a circular race of 1200 m length, A and B start with speeds of 18kmph and 27 kmph starting at the same time from the same point. When will they meet for the first time at the starting point when running in the same direction and opposite direction?
 a) 240sec, 160 sec b) 480sec, 480sec c) 240sec, 240sec d) 160sec, 480sec
13. Two people A and B started running from a same point on a circular track of length 400m at the speed of 8 m/s and 5 m/s respectively. Find the number of meeting points while moving in same directions and in opposite directions.
 a) 5 times, 12 times b) 13 times, 13 times c) 13 times, 10 times d) 15 times, 15 times
14. Three people A, B and C started running from a same point on a circular track of length 400m at the speed of 8 m/s, 5 m/s and 10 m/s respectively. Find the number of meeting points while A and B are moving in same direction and C is moving in opposite direction.
 a) 5 times b) 6 times c) 3 times d) 4 times

PRACTISE QUESTIONS

1. A can run 20 m while B runs 25 m. In a km race, B beats A by:
 - a) 125m
 - b) 200m
 - c) 225m
 - d) 250m
2. In 100m race, A covered a distance in 36 seconds and B covered the same distance in 45 seconds. By what distance did A win the race?
 - a) 25m
 - b) 20m
 - c) 22m
 - d) 18m
3. A can run 1 km in 4min 50 sec and B in 5 min. How many meters start A give B in a km race so that the race may end in a dead heat?
 - a) $500/3$ m
 - b) $400/3$ m
 - c) $100/3$ m
 - d) $200/3$ m
4. In a 100m race, A can beat B by 25m and B can beat C by 4m. In the same race, A can beat C by
 - a) 29m
 - b) 21m
 - c) 26m
 - d) 28m
5. In a 100m race, A beats B by 10m and C by 13m. By what distance B will beat C in a race of 180m.
 - a) 4.5m
 - b) 5.4m
 - c) 6m
 - d) 7.5m
6. In a game of 100 points, A can give B 20 points and C 28 points. How many points that B can give C.
 - a) 8 points
 - b) 10 points
 - c) 12 points
 - d) 14 points
7. Two bike riders ride in opposite directions around a circular track, starting at the same time from the same point. Biker A rides at a speed of 3 kmph and biker B rides at a speed of 1 kmph. If the track has a diameter of 4 km, after how much time (in hours) will the two bikers meet?
 - a) $\frac{22}{7}$ hrs
 - b) $\frac{44}{7}$ hrs
 - c) $\frac{88}{7}$ hrs
 - d) $\frac{11}{7}$ hrs
8. A, B and C run around a circular track of length 2400 meters with respective speeds 9, 18, 27 kmph. If they started at the same time from the same point and run in the same direction When will they meet for the first time at the starting point?
 - a) 16 mins
 - b) 12 mins
 - c) 8 mins
 - d) 14 mins
9. Three people A, B and C run around a circular track of length 400 metre starting simultaneously from the same point in the same direction at 5m/sec, 8 m/sec and 3 m/sec respectively. When will they meet for the first time and when will they meet for the first time at the starting point?
 - a) 400 sec, 400 sec
 - b) 450 sec, 500 sec
 - c) 500 sec, 550 sec
 - d) 300 sec, 400 sec
10. A and B walk round a circular track They start at 5:30 am from the same point in opposite directions A and B walk at a speed of 2 rounds per hour and 3 rounds per hour respectively. How many times shall they cross each other before 7:00 am?
 - a) 5 times
 - b) 6 times
 - c) 7 times
 - d) 8 times

21. PROBLEMS ON TRAINS

Problems on trains can be classified into four categories

✓ **Train passing a stationary point (pole / man / lamp post)**

Length of pole / man / lamp post is negligible while comparing to the length of the train.

Distance = Length of the train.

Speed = Speed of the train.

$$\text{Time} = \frac{\text{Length of the train}}{\text{Speed of the train}}$$

✓ **Train passing a stationary length (platform / bridge)**

Distance = Length of the train + Length of the platform / bridge

Speed = Speed of the train.

$$\text{Time} = \frac{\text{Length of the train} + \text{Length of the platform / bridge}}{\text{Speed of the train}}$$

✓ **Train passing a moving point (man running in same/opposite direction with respect to train)**

Distance = Length of the train

Relative speed = $S_1 + S_2$ (opposite)

Relative speed = $S_1 - S_2$ (same)

$$\text{Time} = \frac{\text{Length of the train}}{\text{Relative speed}}$$

✓ **Train passing a moving length (another train passing in the same/opposite direction)**

Distance = Length of the train 1 + Length of the train 2

Relative speed = $S_1 + S_2$ (opposite)

Relative speed = $S_1 - S_2$ (same)

$$\text{Time} = \frac{\text{Length of the train 1} + \text{Length of the train 2}}{\text{Relative speed}}$$

DRILL

1. A train 110 m long travels at 60 km/hr. How long does it take to cross
 - a) A telegraph post?
 - b) A platform 240 m long?
 - c) A man running at 6 km/hr in the opposite direction?
 - d) A man running at 6 km/hr in the same direction?
 - e) Another train 170 m long, running at 54 km/hr in the same direction?
 - f) Another train 170 m long, running at 80 km/hr in the opposite direction?

2. Two trains are running in opposite direction from the same point. The Speed of the trains is 8 km/hr and 10 km/hr respectively. Find the distance between them after $2\frac{1}{2}$ hours.

- Two trains are travelling in the same direction with the speed of 36 km/hr and 54 km/hr respectively. The faster train overtakes the slower one in 60 sec. What is the length of the either train if they are equal?
- A 100 m long passenger train moving with speed of 50 km/hr crosses another 120 m long train coming from opposite direction in 6 seconds. What is the speed of the second train?
- A train crosses a pole in 10 sec and a 150 m long platform in 16 sec. What is the speed of the train in km/hr?
- Two trains start at the same time from two stations 360 km apart. If the trains meet after 5 hours and difference between their speeds is 12 km/hr, find the speed of the faster train.
- A train leaves station A towards B at an average speed of 80 km/hr. After 3 hours another trains leaves station A at an average speed of 110 km/hr. Find the distance from station A where the two trains meet.

PRACTISE QUESTIONS

- A train covers 85 m in passing a signal post. What is the length of the train?
- A 29 m long train passes a platform which is 100 m long. Find the distance covered by the train passing the platform.
- A 250 m long train is running at 100 km/hr. In what time, will it pass a man running at 10 km/hr in the same direction in which the train is going?
- A 220 m long train is running at 120 km/hr. In what time will it pass a man running in the direction opposite to that of the train at 12 km/hr speed?
- Two trains of length 80 m and 90 m are moving in opposite direction at 10 m/s and 7 m/s respectively. Find the time taken by the train to cross each other.
- Two trains of length 75 m and 95 m are moving in opposite direction at 9 m/s and 8 m/s respectively. Find the time taken by the faster train to cross the slower train.
- A train passes a standing man in 6 s and a 210 m long platform in 16 s. Find the length and the speed of the train.
- Two trains start at the same time from stations A and B towards each other. Distance between the two stations is 420 km and the trains meet each other after 6 hours. If the difference between the speed of the trains is 10 km/hr, find the speed of the slower train.
- Two trains start at the same time from points A and B towards each other, and after crossing each other, they take 25 hr and 9 hr in reaching points B and A respectively. Find the ratio of speeds of first train to that of second train.

22. BOATS AND STREAMS

Problems based on boats and streams deals with the travel of boat in water stream.

Still water:

If the speed of water in the river is zero, then that water is consider to be still water.

Stream water:

If water in the river is moving at certain speed, then that water is called as stream water.

Downstream:

When a boat is moving along or with the stream is called downstream.

Upstream:

When a boat is moving against or opposite to the stream is called upstream.

Basic formulae related to boats and streams:

If the speed of boat in still water is x km/hr and speed of the stream is y km/hr, then

- ✓ Downstream Speed $D_s = x + y$
- ✓ Upstream Speed $U_s = x - y$
- ✓ Speed of boat in still water $x = \frac{D_s + U_s}{2}$
- ✓ Speed of the stream $y = \frac{D_s - U_s}{2}$

DRILL

1. The speed of the boat in still water is 15 km/hr. If the speed of the stream is 3 km/hr, then find the downstream speed & upstream speed.
2. If a man can row downstream at 16 km/hr and upstream 10 km/hr, then find his speed in still water & speed of the stream.
3. A boat takes 8 hr to row 48 km downstream and 12 hr to row the same distance upstream. Find the boat speed in still water and rate of the current.
4. If a man can row a boat 15 km/hr with the current and speed of current is 3 km/hr, then find his speed against the current and speed of boat in still water.
5. What time will be taken by a boat to cover a distance of 64 km along the stream if the speed of boat in still water is 12 km/hr and speed of stream is 4 km/hr?
6. A man can swims 3 km/hr in still water. If the velocity of the stream is 2 km/hr, then find the time taken by him to swim to a place 10 km upstream and back.
7. A man rows a boat in still water in 10 km/hr. If takes 5 hours to row a certain distance and return to the starting point in a river flowing at 4 km/hr. Find the distance between the two points.
8. A boat's speed in still water is 5 km/hr, while river is flowing with the speed of 2km/hr and time taken to cover a distance upstream in 2 hours more than time taken to cover the same distance downstream. Find the distance.

9. Siva can row 12 km/hr in still water. It takes him twice as long to row-up as to row-down the river. Find the rate of stream.
10. Covering a distance of 36 km in both against and along the stream at the speed of 10 km/hr in still water. If it gives 90 minutes difference, then find the speed of the current in (km/hr).

PRACTISE QUESTIONS

1. The speed of a boat in still water is 8 km/hr and the rate of stream is 4 km/hr. Find the upstream & downstream speed of the boat.
2. A person swims downstream 64 km in 4 hours and 40 km upstream in 5 hours. Find the speed of the stream & his speed in still water.
3. What time will be taken by a boat to cover a distance of 128 km along the stream if the speed of boat in still water is 24 km/hr and speed of stream is 8 km/hr?
4. The speed of a boat in still water is 12 km/hr and speed of the stream is 3 km/hr. The total time taken to travel A to B and back is 8 hours. Find the distance between A and B.
5. A boat's speed in still water is 10 km/hr, while river flowing with a speed of 2 km/hr and time taken to cover certain distance upstream is 4 hr more than the time taken to cover the same distance downstream. Find the distance.
6. The speed of a boat in still water is 30 km/hr. If it takes thrice as long to go upstream to a point as to return downstream to the starting point. What is the rate of current?
7. Kamala can row a certain distance downstream in 12 hr and can return the same distance in 18 h. If stream flows at the rate of 6 km/hr, then find the speed of kamala in still water?
8. Rajesh rows in still water with a speed of 4.5 km/hr to go to a certain place and to come back. Find average speed for whole journey, if river is flowing with a speed of 1.5 km/hr.
9. A boat whose speed in still water is 8 km/hr, takes 5 hours to go 15 km upstream and 22 km downstream. Find the speed of the stream.
a) 3 b) 5 c) 2.5 d) 11
10. In a river flowing at a 2 km/hr, a boat travels 32 km upstream and then returns downstream to starting point. If speed in still water be 6 km/hr, find total journey time.
11. A motorboat travelling at some speed can cover 25 km upstream and 39 km downstream in 8 hr. At the same speed it can travel 35 km upstream and 52 km downstream in 11 hr, find the speed of the stream.
a) 2 b) 3 c) 4 d) 5
12. A man rows 6 hours downstream and then 6 hours upstream. In this whole journey he covers a total distance of 24 kms. If the speed of the stream is 1 kmph, for how much more time will he have to row upstream to reach the starting point?

23. PERMUTATION

Permutation

Permutation implies number of possible ways to arrange certain number of things, where order of the things is important. Each of the different arrangements can be made by taking some or all of a given number of things or objects at a time.

For Example,

The permutations of three elements A, B and C taken two at a time are AB, BA, BC, CB, AC and CA. Since, the order in which the items are taken, AB and BA are counted as two different permutations.

Types of permutation

- ✓ Linear permutation $n_p r = \frac{n!}{(n-r)!}$
- ✓ Circular permutation $(n-1)!$
- ✓ Distribution of things (n^r)

Example 1:

In how many ways A, B, C can be arranged.

Solution:

$$n! = 3! = 6 \text{ ways} \quad (\text{since, here } n = r)$$

Example 2:

In how many ways A, B, C, D can be arranged in 2 spaces.

Solution:

Here $n = 3, r = 2$

$${}^4C_1 \times {}^3C_1 = 4 \times 3 = 12 \text{ ways}$$

Example 3:

In how many ways A, B, C can be arranged if repetition is allowed.

Solution:

$${}^3C_1 \times {}^3C_1 \times {}^3C_1 = 3 \times 3 \times 3 = 27 \text{ ways}$$

Example 4:

In how many ways A, A, A, B, B, C can be arranged in 3 spaces.

Solution:

$$\frac{{}^6C_1 \times {}^5C_1 \times {}^4C_1}{3! \times 2!} = \frac{6 \times 5 \times 4}{(3 \times 2) \times 2} = 10 \text{ ways}$$

Example 5:

In how many ways A, A, A, B, B, C can be arranged in 3 spaces if repetition is allowed.

Solution: $\frac{{}^6C_1 \times {}^6C_1 \times {}^6C_1}{3! \times 2!} = \frac{6 \times 6 \times 6}{(3 \times 2) \times 2} = 18 \text{ ways}$

DRILL**A) WORDS****NON-REPEATED LETTERS IN THE GIVEN WORD**

1. In how many ways the following words can be formed
PILOT , WATCH, CAR, ANSWER
2. In how many four letter words can be formed from the word WORLD.

REPEATED LETTERS IN THE GIVEN WORD

3. In how many ways following words can be arranged
LAPTOP, APTITUDE, MISSISSIPPI.
4. How many six letter words can be formed from the word “MALAYALAM”?

REPETITION ALLOWED

5. In how many ways A, B, C, D, E can be arranged, if repetition is allowed.
6. In how many ways following words can be arranged when repetition is allowed
BAG, CANDY, SYSTEM, APTITUDE, ENHANCE, TECHNOLOGY.

ALWAYS TOGETHER

7. In how many different ways can the letters of the word LEARN can be arranged so that the vowels always come together?
8. In how many different ways can the letters of the word LEARN can be arranged so that the consonants always come together?
9. In how many different ways can the letters of the word APTITUDE can be arranged so that the vowels always come together?
10. In how many different ways can the letters of the word LEGAL can be arranged so that the consonants always come together?

NEVER TOGETHER

11. In how many different ways can the letters of the word LEARN can be arranged so that the vowels never come together?
12. In how many different ways can the letters of the word LEARN can be arranged so that the consonants never come together?
13. In how many different ways can the letters of the word WINDOW can be arranged so that the vowels never come together?
14. In how many different ways can the letters of the word VALUE can be arranged so that the consonants never come together?

OCCUPYING ODD/EVEN PLACES

15. In how many different ways can the letters of the word 'APTITUDE' be arranged in such a way that the vowels occupy only the even positions?
16. In how many different ways can the letters of the word FRIDGE be arranged in such a way that the vowels occupy only the odd positions?

B) NUMBERS**NON-REPEATED NUMBERS IN GIVEN**

1. How many three digit number can be formed using 1, 2, 3, 4.
2. How many two digit number can be formed using 1, 2, 3, 4.

REPEATED NUMBERS IN GIVEN

3. How many two digit numbers can be formed from 1, 2, 2, 3, 4.
4. How many three digit number can be formed from 7, 9, 8, 9, 4, 9.

ZERO IN THE GIVEN NUMBER

5. How many four digit numbers can be formed using 0,1,2,3.
6. How many three digit number can be formed from 8, 9, 0.

REPETITION ALLOWED

7. How many three digit numbers can be formed using 1, 2, 3 if repetition is allowed.
8. How many two digit numbers can be formed using 1, 2, 3 if repetition is allowed.

MISCHELLANEOUS

9. How many 3 digit numbers can be formed using 2, 3, 5, 6, 7 and 9, which are divisible by 5 and none of the digits is repeated?
10. How many integers greater than 999 but not greater than 4000 can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed?

PRACTICE QUESTIONS

1. How many three digit number can be formed from three different digits.
2. How many four digit numbers can be formed from 5, 7, 6, 8, 3.
3. How many two digit numbers can be formed from 4, 2, 3, 4.
4. How many three digit numbers can be formed from 1, 5, 2, 3, 6, 4, 5, 6.
5. How many five digit numbers can be formed using 1, 0, 3, 2, 0.
6. How many three digit numbers can be formed from 1, 5, 3, 0.
7. How many five digit passwords are possible using the numbers 2, 4, 5, 6, 8.
8. How many four digit numbers can be formed from 1 and 2.

CIRCULAR PERMUTATION

1. 20 persons are invited. In how many ways can they and the host be seated in a circular table?
2. In how many different ways 5 girls and 5 boys can sit on a circular table, such that the boys and the girls are alternate?
3. Find the number of ways, in which 12 different beads can be arranged to form a necklace.

DISTRIBUTION OF THINGS

1. In how many ways, 3 books can be given away to 7 boys, when each boy is eligible for any of the books?
2. In how many ways a man can invite his 6 friends, if he has 3 servants to deliver the invitation?
3. A child has four pockets and three marbles. In how many ways the child can put the marbles in the pocket?

24. COMBINATION

Combination

Combination implies number of possible ways to select certain number of things. Here, order of things has no importance.

For Example,

The combination of two letters from the group of three letters A, B and C would be as follows AB, BC and AC. Here, we make groups. So, AB or BA as a group is same. Obviously, if order matters, then AB and BA are not same. It signifies number of groups formed from n different things, when r things are taken into consideration.

Important Points

- ✓ $n_{C_1} = n$
- ✓ $n_{C_n} = 1$
- ✓ $n_{C_0} = 1$
- ✓ $n_{C_r} = n_{C_{n-r}}$
- ✓ $0! = 1$

DRILL

1. In how many ways can 11 cricket players be chosen out of a batch of 15 players?
2. In a class, there are 20 boys and 10 girls. In how many ways a teacher can select either a boy or a girl to represent?
3. From a group of 5 boys and 4 girls, 3 boys and 2 girls are required to be selected. In how many ways can they be chosen?
4. In how many ways can 5 members form a committee out of 10 be selected so that,
 - a) Two particular members must be included
 - b) Two particular members must not be included
5. A hall has 12 gates. In how many ways can a man enter the hall through one gate and come out through a different gate?
6. In a cricket tournament 5 matches were played, then in how many ways result can be declared?
7. In an exam a minimum mark is to be scored in each six subjects to pass. In how many ways can a student fail?

PRACTICE QUESTIONS

1. In how many ways a committee consisting of 5 men and 6 women can be formed from 8 men and 10 women?
2. In how many ways a cricket team of 11 players can be made from 15 players, if a particular player is always chosen?
3. In how many ways a cricket team of 11 players can be made from 15 players, if a particular player is never chosen?

4. There are three stations A,B and C and five routes for going from station A to B and four routes from station B to C. Find the number of different ways through which a person can go from A to C via B.
5. Find the number of ways in which a mixed double game can be arranged from amongst 9 married couples if no husband and the wife play in the same game.
6. A box contains 20 electric bulbs, out of which 4 are defective. Find the probability that atleast one is defective, if two bulbs are chosen at random from this box.

PERMUTATION AND COMBINATION

1. Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
2. How many ways can necklaces of 4 beads can be made from 12 beads?
3. In How many ways can 24 persons be seated around a circular table if there are 13 seats?
4. How many necklaces of 12 beads can be made from 18 beads of various colours?
5. If the number of words formed with or without meaning with each of 2 vowels and 3 consonants taken from the letters of the word "EVOLUTION" is written in the form $2^a \times 3^b \times 5^c \times 13^d$, then find $a + b + c + d$.

HANDSHAKE PROBLEMS

- ✓ If handshake is among ‘n’ number of people, then the number of handshake will be,
 - a) LINEAR ARRANGEMENT
Number of handshake = $(n-1)$ handshakes
 - b) CIRCULAR ARRANGEMENT
Number of handshake = (n) handshakes
 - c) GROUP OF PEOPLE
Number of handshake = $[n(n-1)/2]$ handshakes
- ✓ If ‘n’ number of people in a group, number of gifts given among everyone = $n(n-1)$ gifts.

DRILL

1. There are 20 people in a party. If everyone is to shake hands with one another, how many handshakes are possible?
2. In a party every person shakes hands with every other person. If there are 105 hands shakes, find the number of person in the party.
3. In a party every person shakes his hands with every other person. If there are 210 hands shakes, find the number of person in the party.
4. In a meeting between two countries, each country has 12 delegates, all the delegates of one country shakes hands with all delegates of the other country. Find the number of handshakes possible?

25. PROBABILITY

Probability means the chances of happening or occurring of an event. When an event is certain to happen then the probability of occurrence of that event is 1 and when it is certain that the event cannot happen then the probability of that event is 0. Hence the value of probability ranges from 0 to 1. Probability of certain event is 1 and Probability of impossible event is 0.

$$\text{Probability (P)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Sample space

A sample space of an experiment is the set all possible outcomes of that experiment. It is denoted by S.

For example,

Sample space of tossing n coin has 2^n

Sample space of throwing n dice is $= 6^n$

i.e., If we throw a die, then sample space $S = \{1, 2, 3, 4, 5, 6\}$

If we toss a coin, then sample space $S = \{\text{Head, Tail}\}$

Possible outcomes

All possibilities related to an event are known as possible outcomes.

For example, when a coin is tossed, there are two possible outcomes (Head, Tail). So, the probability of getting H is $1/2$ or the probability of getting T is $1/2$,

Similarly, when a single die is thrown, there are six possible outcomes 1, 2, 3, 4, 5 and 6. So, the probability of getting 4 is $1/6$.

DRILL

COINS

1. In a throw of a coin, find the probability of getting a head.
2. Two unbiased coins are tossed simultaneously. Find probability of getting atmost 1 head.
3. Three unbiased coins are tossed. What is the probability of getting atleast 2 heads?

DICE

4. An unbiased die is tossed. Find the probability of getting a multiple of 3.
5. In a simultaneous throw of pair of dice, find the probability of getting 3 on first die & an even number on the second die.
6. A pair of dice is rolled. What is the probability of getting a total of 2 or 5?
7. A die is thrown twice. Find probability that atleast one of them comes up with number 4?
8. In a simultaneous throw of 2 dice, what is the probability of getting a doublet?

MARBLES / BALLS

Directions (9 -16): A basket contains 4 red, 5 blue and 3 green balls

9. What is the probability that it is blue when one ball is picked up randomly?
10. If one ball is picked up randomly, what is the probability that it is neither red nor blue?
11. If two balls are picked at random, what is the probability that both are red?
12. If two balls are picked at random, what is the probability that either all are green or all are red?
13. If 4 balls are picked at random, what is the probability that 2 balls are red and 2 are green?
14. If three balls are picked at random, what is the probability that none is blue?
15. If three balls are picked at random, what is the probability that atleast one is blue?
16. If three balls are picked at random, what is the probability that atmost 2 is green?
17. A bag contains 4 blue balls, 3 red balls and 5 grey balls. If two balls are drawn at random, what is the probability that both of them are of the same colour?
18. A bag contains 4 green, 5 red and 7 white balls. Three balls are drawn randomly, what is the probability that all the three drawn balls are of different colour?
19. A box contains 5 green, 4 yellow and 3 white marbles. Three marbles are drawn at random. What is the probability that they are not of the same colour?

CARDS

20. One card is drawn from a pack of 52 cards. What is the probability of getting a queen?
21. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card?
22. One card is drawn from a pack of 52 cards. What is the probability of getting a queen of club or a king of heart?
23. What is the probability that a card drawn from a pack of 52 cards will be a diamond or a king?
24. One card is drawn from a pack of 52 cards. What is the probability that the card drawn is either a red card or a king?

25. From a pack of 52 cards, two cards are drawn together at random. What is the probability for both the cards being king?
26. Two cards are drawn together from a pack of 52 cards. What is the probability that one is a spade and one is a heart?
27. Two cards are drawn from a pack of 52 cards. What is the probability that either both are jack or both are ace?
28. Two cards are drawn from a pack of 52 cards. What is the probability that either both are red or both are king?

MISCHELLANEOUS

29. A single letter is selected at random from the word 'PROBABILITY'. What is the probability that it is a vowel?
30. What is the probability that a leap year selected at random contains 53 Sundays?
31. Let E be the set of all integers with 1 at their unit place. What is the probability that a number chosen from $\{2, 3, 4, \dots, 50\}$ is an element of E?
32. A committee of 3 members to be selected out of 3 men and 2 women. What is the probability that the committee has atleast one woman?
33. Ticket numbered 1 to 20 are mixed up and then a ticket a drawn at random. What is the probability that the ticket drawn has a number which is a multiple of 3 or 5?
34. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?
35. Four persons are chosen at random from a group of 3 men, 2 women and 4 children. What is the chance that exactly 2 of them are children?
36. A box contains 20 electric bulbs out of which 4 are defective. Two bulbs are chosen at random from this box. What is the probability that atleast one of these is defective?
37. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi?
38. A speaks truth in 75% cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other, narrating the same incident?
39. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is $(1/7)$ and the probability of wife's selection is $(1/5)$. What is the probability that only one of them is selected?

40. A person has 2 bags. He has 3 black and 4 white balls in one bag and 4 black and 3 white balls in another bag. Find the probability of getting a black ball.
41. In a mathematics class of 30 students, 17 are boys and 13 are girls. On a unit test, 4 boys and 5 girls made an A grade. If a student is chosen at random from the class, what is the probability of choosing a girl or an A grade student?
42. The probability that a man will be alive for 10 more years is $\frac{1}{4}$ and the probability that his wife will be alive for 10 more years is $\frac{1}{3}$. What is the probability that none of them will be alive for 10 more years?
43. Two persons A and B appear in an interview for two vacancies. If the probabilities of their selection are $\frac{1}{4}$ and $\frac{1}{6}$ respectively, then what is the probability that none of them is selected?
44. There are 6 boys and 6 girls. They sit in a row randomly. What is the probability that all the girls sit together?

26. DATA INTERPRETATION

TABLE CHARTS

- I. Directions (Q.No. 1 to 5): Study the following table carefully and answer the given questions.

Production of five types of cars by a company in the years 1989 to 1994

Year Type	1989	1990	1991	1992	1993	1994	Total
P	8	20	16	17	21	6	88
Q	16	10	14	12	12	14	78
R	21	17	16	15	13	8	90
S	4	6	10	16	20	31	87
T	25	18	19	30	14	27	133
Total	74	71	75	90	80	86	476

1. In which year, the production of cars of all types taken together was approximately equal to the average of the total production over the years?
 a) 1989 b) 1991 c) 1993 d) 1994

2. In which year, the total production of cars of types P and Q together was equal to the total production of cars of types R and S together?
 a) 1990 b) 1991 c) 1994 d) 1993

3. During the period 1989-94, which type of cars had a continuous increase in the production?
 a) P b) Q c) R d) S

4. The production of which type of car was 25% of the total production of all types of cars during 1993?
 a) P b) R c) Q d) S

5. The percent increase in total production of all types of cars in 1992 to that in 1991 was
 a) 15% b) 20% c) 25% d) 30%

- II.** Directions (Q.No. 6 to 10): Study the following table carefully and answer the questions given below.

Number of students studying five different disciplines from five institutes

Discipline/ Institute	Discipline				
	Art	Commerce	Science	Management	Computer Science
A	350	260	450	140	300
B	240	320	400	180	320
C	460	300	360	160	380
D	440	480	420	120	340
E	280	360	340	200	330

6. Find the sum of students studying in Arts in institute A, Commerce in institute B, Science in institute C, Management in institute D, Computer Science in institute E.
 a) 1480 b) 1560 c) 1460 d) 1580

7. What is the average number of students studying Commerce from all the institutes together?
 a) 356 b) 360 c) 348 d) 344

8. Total number of students studying Art from institutes A and B together is approximately what percent of the total number of students studying Computer Science from these two institutes?
 a) 84% b) 85% c) 95% d) 90%

9. Number of students studying Commerce from institute D is what percent of the total number of students studying all the disciplines together from this institute?
 a) $28\frac{1}{3}\%$ b) $26\frac{2}{3}\%$ c) $24\frac{2}{3}\%$ d) $24\frac{1}{3}\%$

10. What is the ratio of the total number of students studying Science from institutes C and D together to the total number of students studying Computer Science from these two institutes together?
 a) 13:12 b) 12:13 c) 13:15 d) 15:13

III. Directions (Q.No. 11 to 15): Study the following table carefully and answer the questions given below.

**Number of tickets sold in a week of five movies in the multiplexes in six different cities
(in thousands)**

Movie City	A	B	C	D	E
Mumbai	20	15	35	26	18
Delhi	17	19	21	25	28
Kolkata	32	24	19	21	17
Chennai	18	21	32	28	34
Hyderabad	16	34	26	29	22
Lucknow	15	27	20	35	26

11. The number of tickets of movie B sold in Hyderabad was approximately what percent of the total number of tickets of the same movie sold in all the cities together?
 a) 15 b) 24 c) 18 d) 20

12. What is the difference between the number of tickets of movie D sold in Kolkata and the number of tickets of movie B sold in Lucknow?
 a) 700 b) 14000 c) 6000 d) 9000

13. What is the average number of tickets of movie C sold in all the six cities?
 a) 15500 b) 2550 c) 24000 d) 25500

14. The number of tickets of movie E sold in Chennai is what percent of number of tickets of movies A sold in Mumbai?
 a) 170% b) 70% c) 30% d) 130%

15. In which city was the total number of tickets of all the five movies together sold the minimum?
 a) Delhi b) Chennai c) Lucknow d) Kolkata

- IV. Directions (Q.No. 16 to 20): Read the following table and answer the questions that follow.

**Number of Different Types of Batteries Sold by a Company Over the Years
(Numbers in Thousands)**

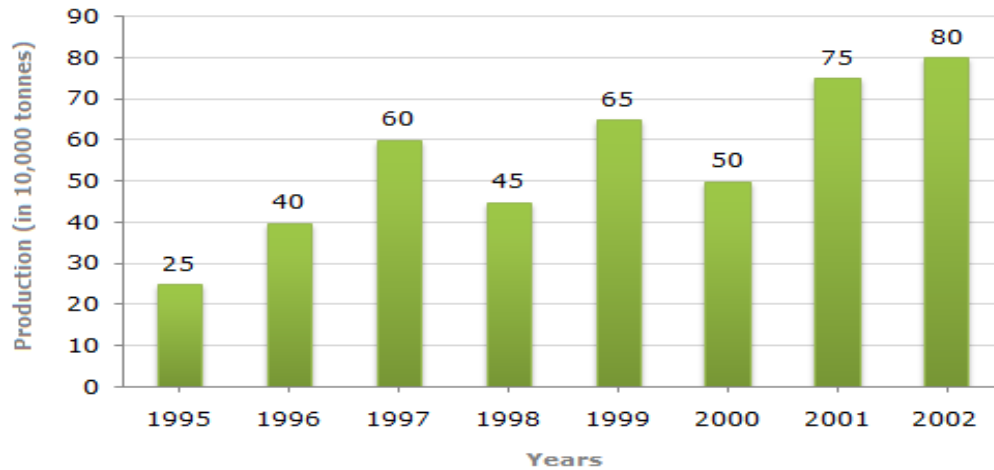
Year	Types of Batteries					
	4Ah	7Ah	32Ah	35Ah	55Ah	Total
1992	75	144	114	102	108	543
1993	90	126	102	84	126	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

16. What was the approximate percentage increase in the sales of 55Ah batteries in 1998 compared to that in 1992?
 a) 28% b) 31% c) 33% d) 34%
17. The total sale over the years was the maximum for which battery?
 a) 4Ah b) 7Ah c) 32Ah d) 35Ah
18. What is the difference in the number of 35AH batteries sold in 1993 and 1997?
 a) 24000 b) 28000 c) 35000 d) 39000
19. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year?
 a) 1994 b) 1995 c) 1996 d) 1997
20. In case of which battery there was a continuous decrease in sales from 1992 to 1997?
 a) 4Ah b) 7Ah c) 32Ah d) 35Ah

BAR CHARTS

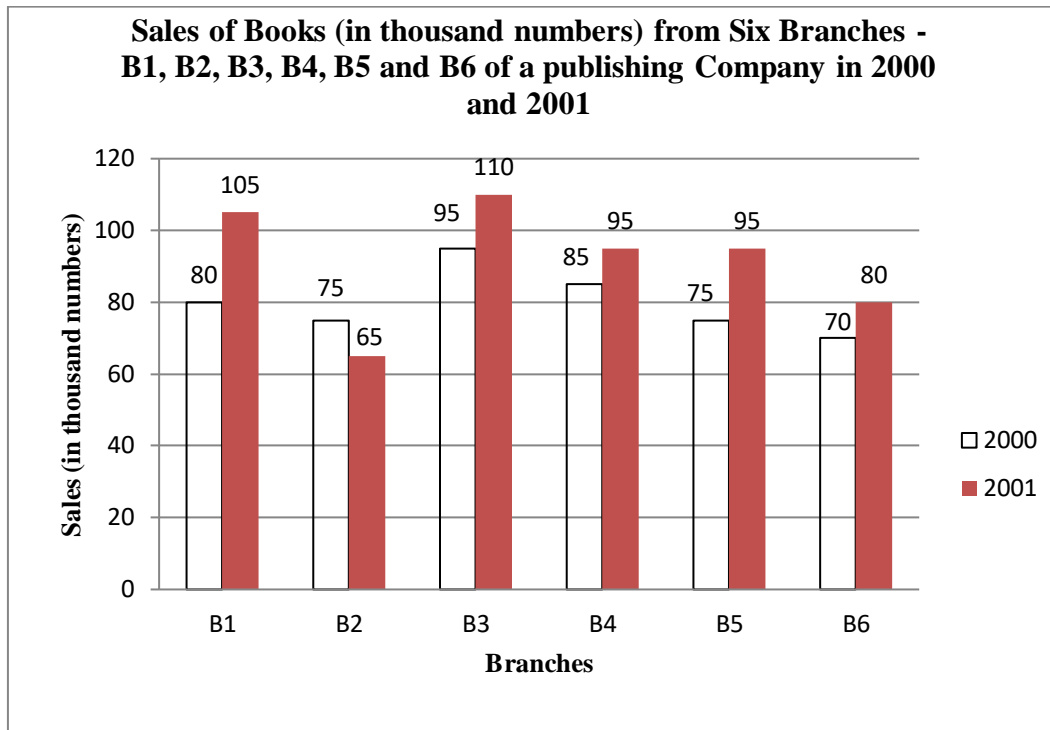
- I. Directions (Q.No. 1 to 5): Study the bar chart and answer the question based on it.

Production of Fertilizers by a Company (in 10000 tonnes) Over the Years



- What was the percentage decline in the production of fertilizers from 1997 to 1998?
 a) 33.33% b) 20% c) 25% d) 21%
- The average production of 1996 and 1997 was exactly equal to the average production of which of the following pairs of years?
 a) 2000 & 2001 b) 1999 & 2000 c) 1998 & 2000 d) 1995 & 2001
- What was the percentage increase in production of fertilizers in 2002 compared to that in 1995?
 a) 320% b) 300% c) 220% d) 200%
- In which year the increase in production as compared to the previous year was maximum?
 a) 2002 b) 2001 c) 1997 d) 1996
- In how many of the given years was the production of fertilizers more than the average production of the given years?
 a) 1 b) 2 c) 3 d) 4

II. Directions (Q.No. 6 to 10): Study the bar chart and answer the question based on it.



6. What is the ratio of the total sales of branch B2 for both years to the total sales of branch B4 for both years?
 a) 2:3 b) 3:5 c) 4:5 d) 7:9

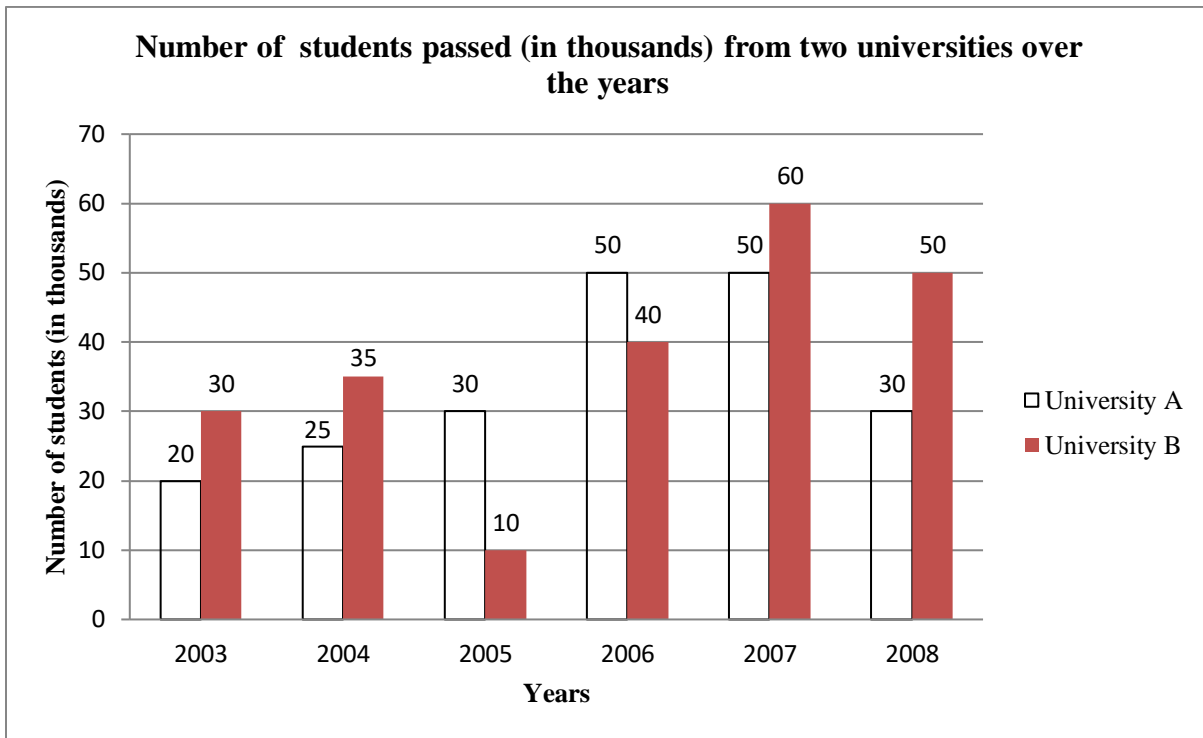
7. Total sales of branch B6 for both the years is approximately what percent of the total sales of branches B3 for both the years?
 a) 69% b) 71% c) 73% d) 76%

8. The sale of branch B6 in 2000 is what percentage of the sale of same branch in 2001?
 a) 75% b) 77.5% c) 82.5% d) 87.5%

9. What is the average sale of all the branches (in thousand numbers) for the year 2000?
 a) 73 b) 80 c) 83 d) 88

10. Find the total sales of branches B1, B3 and B5 together for both the years (in thousand numbers).
 a) 250 b) 310 c) 435 d) 560

III. Directions (Q.No. 11 to 15): Study the following graph carefully and answer the question based on it.



11. What is the respective ratio of the number of students passed from university A in the year 2007 and the number of students passed from university B in the year 2004?
 - a) 5:4
 - b) 4:5
 - c) 7:10
 - d) 10:7

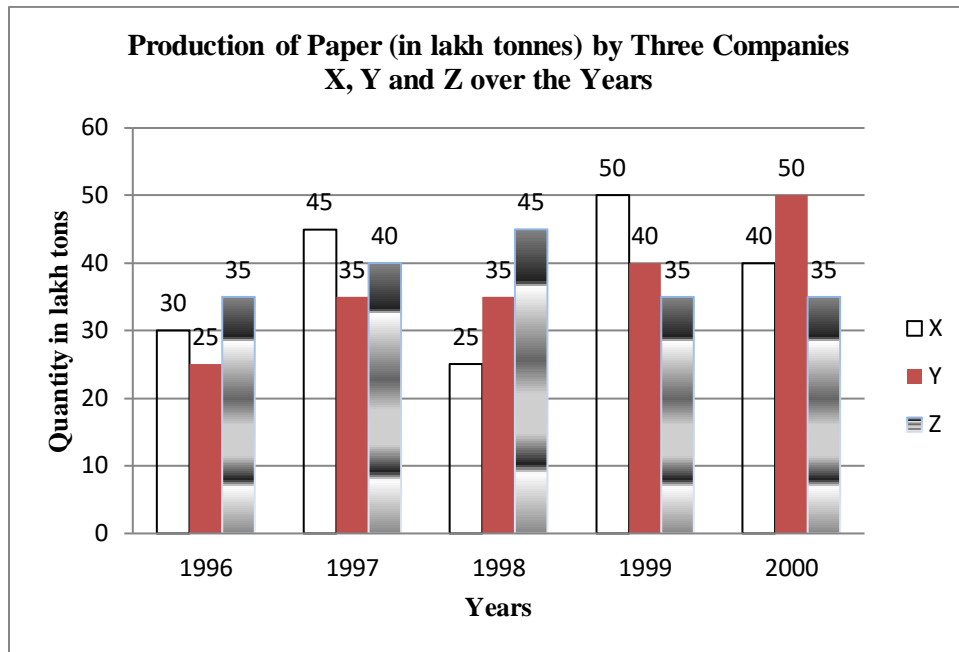
12. What is the difference between the total number of students passed from both the universities in the year 2007 together and the total number of students passed in the year 2005 from both the universities together?
 - a) 70000
 - b) 37000
 - c) 7000
 - d) 3700

13. What is the sum of students passed from university B in the years 2003, 2005 and 2006 together?
 - a) 80000
 - b) 8000
 - c) 800000
 - d) 75000

14. Number of students passed from university B in 2008 is approximately what percent of the total number of students passed from university A over the years?
 - a) 30%
 - b) 25%
 - c) 20%
 - d) 35%

15. What is the respective ratio of the number of students passed in 2007, 2008 and 2005 from university A?
 - a) 5:3:2
 - b) 3:5:5
 - c) 5:3:3
 - d) 5:1:1

IV. Directions (Q.No. 16 to 20): Study the following graph carefully and answer the question based on it.



16. Find the production of y over the years in tonnes.
 - a) 185
 - b) 175
 - c) 195
 - d) 205

17. For which of the following years, the rise in production from the previous year is the maximum for Company X?
 - a) 1999
 - b) 1996
 - c) 2000
 - d) 1998

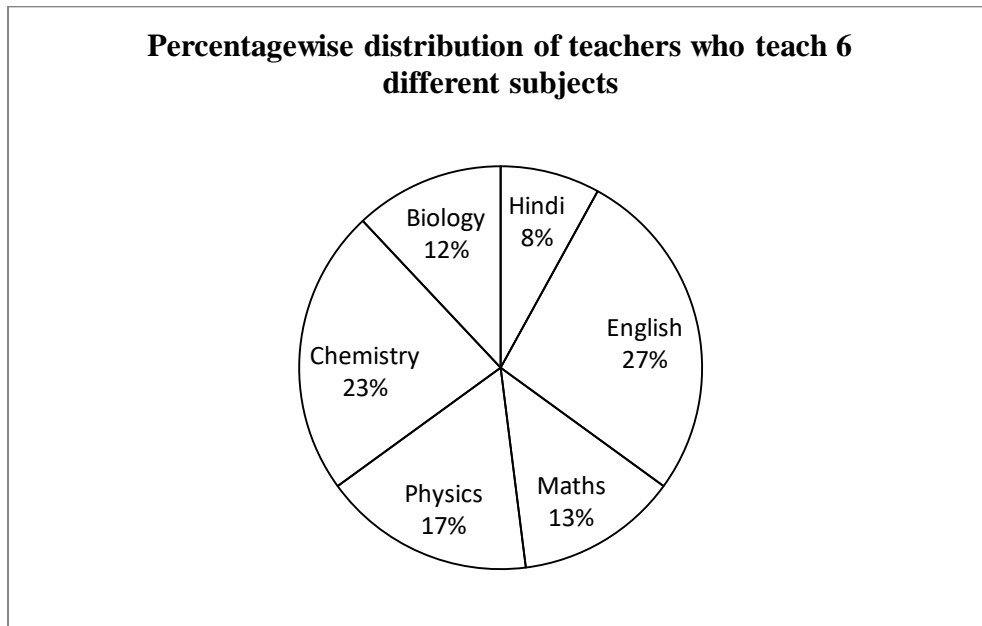
18. What is the ratio of the average production of Company X in the period 1998-2000 to the average production of Company Y in the same period?
 - a) 1:1
 - b) 15:17
 - c) 23:25
 - d) 27:29

19. What is the average production for five years for the companies X and Z?
 - a) 37, 38
 - b) 38, 37
 - c) 38, 38
 - d) 36, 38

20. What is the percentage increase in the production of Company Y from 1996 to 1999?
 - a) 25%
 - b) 60%
 - c) 70%
 - d) 50%

PIE CHARTS

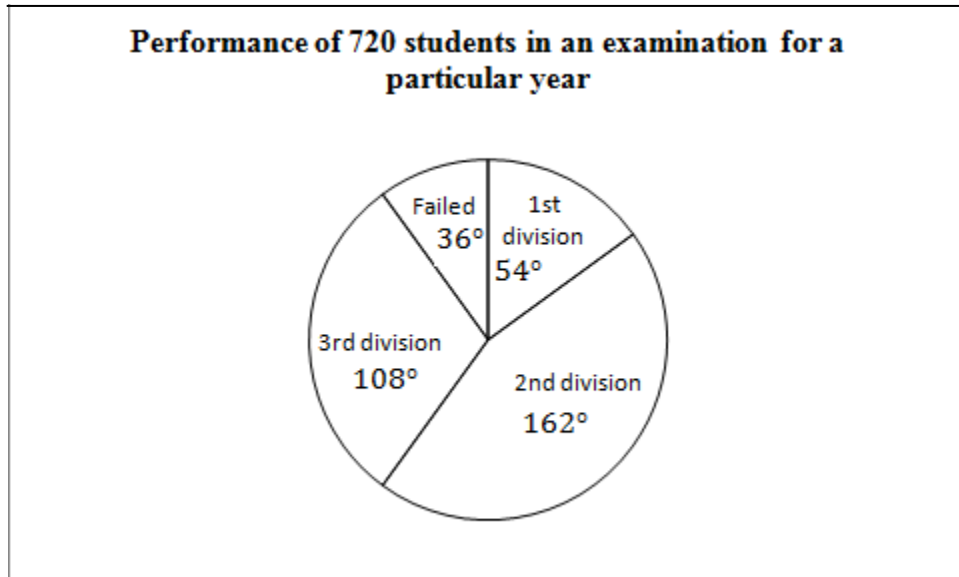
- I. Directions (Q.No. 1 to 5): Study the following pie chart carefully to answer the questions given below.



Total number of teachers = 1800

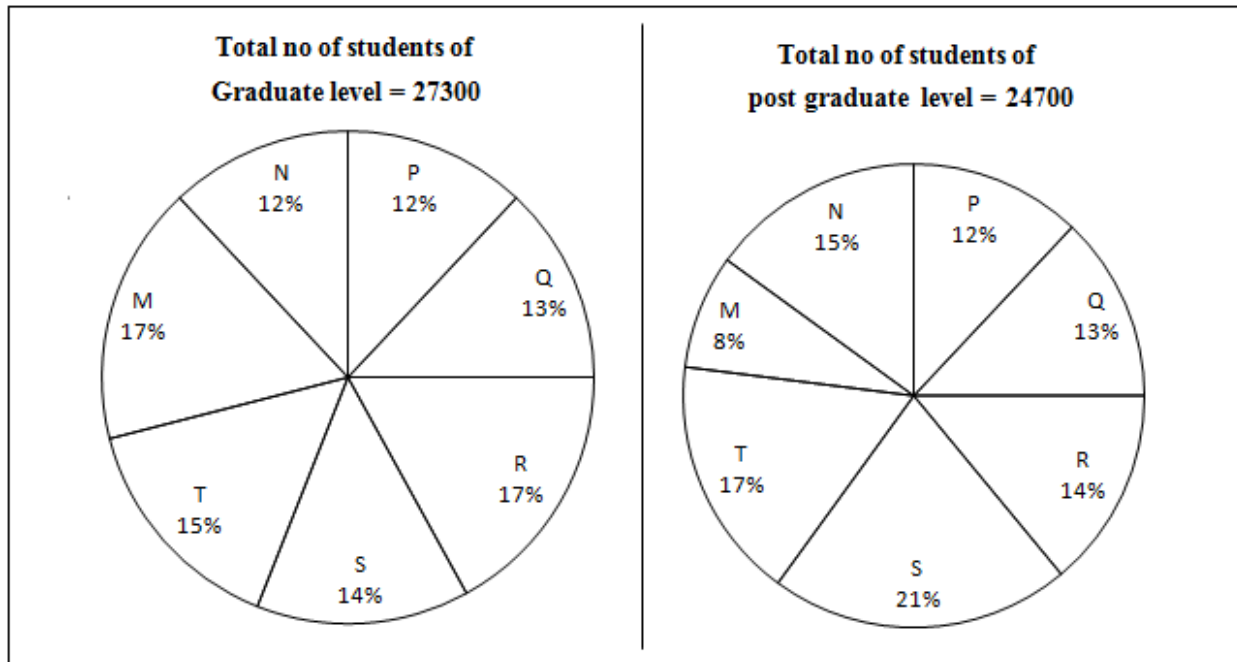
- What is the total number of teachers teaching Chemistry, English and Biology?
 - 1226
 - 1116
 - 1176
 - 998
- What is the difference between the total number of teachers who teach English and Physics together and the total number of teachers who teach Maths and Biology together?
 - 352
 - 342
 - 643
 - 653
- What is the respective ratio of the number of teachers who teach Maths and the number of teachers who teach Hindi?
 - 13:8
 - 7:13
 - 7:26
 - 8:15
- The number of Hindi teachers is approximately what percent of the total number of teachers who teach Biology?
 - 67%
 - 25%
 - 50%
 - 33%
- If the percentage of Maths teachers is increased by 50% and percentage of Hindi teachers is decreased by 25%, then what will be the total number of Maths and Hindi teachers together?
 - 390
 - 379
 - 459
 - 480

II. Directions (Q.No. 6 to 10): Study the pie chart and answer the questions given below.



6. The number of students who passed on 1st division is
 a) 90 b) 108 c) 128 d) 148
7. The number of students who passed in 2nd division is more than those in 1st division by
 a) 222 b) 224 c) 218 d) 216
8. The ratio of successful students to the failed students is
 a) 9:1 b) 5:1 c) 1:9 d) 2:7
9. The percentage of students who have failed in the examination is
 a) 20% b) 36% c) 10% d) 30%
10. The total number of students who have passed in 2nd and 3rd division is
 a) 324 b) 540 c) 216 d) None of the above

III. Directions (Q.No. 11 to 15): Study the pie chart and answer the questions given below.



Distribution of students at graduate and post-graduate levels in seven institutes

11. What is the total number of graduate and post-graduate level students is institute R?
 a) 8320 b) 7916 c) 9116 d) 8099

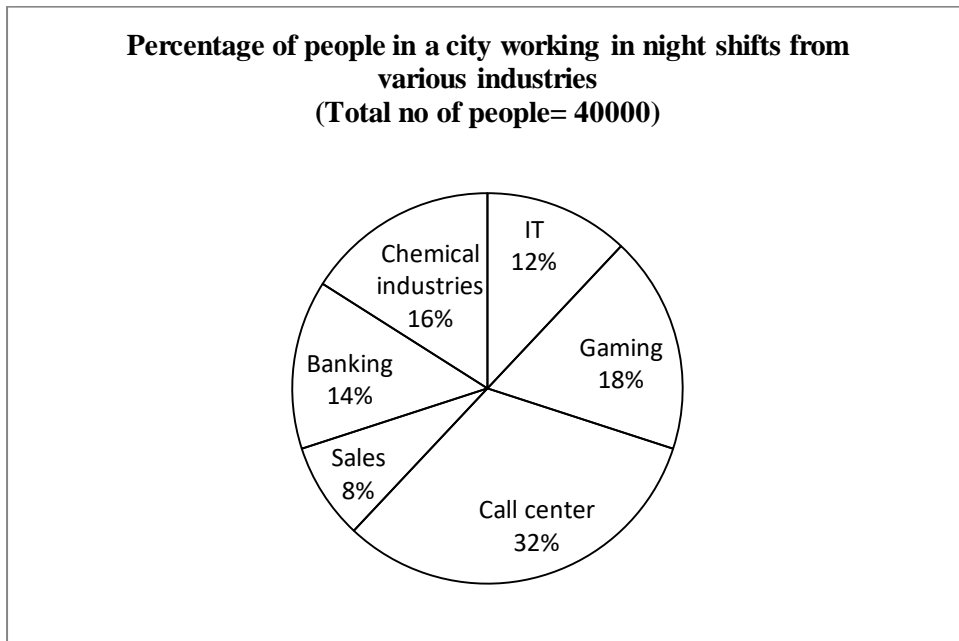
12. What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S?
 a) 14:19 b) 19:21 c) 17:21 d) 19:14

13. How many students of institutes of M and S are studying at graduate level?
 a) 7516 b) 8463 c) 9127 d) 9404

14. What is the ratio between the number of students studying at post-graduate level from institutes S and the number of students studying at graduate level from institute Q?
 a) 13:19 b) 21:13 c) 13:8 d) 19:13

15. Total number of students studying at post-graduate level from institutes N and P is
 a) 5601 b) 5944 c) 6669 d) 8372

IV. Directions (Q.No. 16 to 19): Study the following pie chart carefully to answer the questions given below.



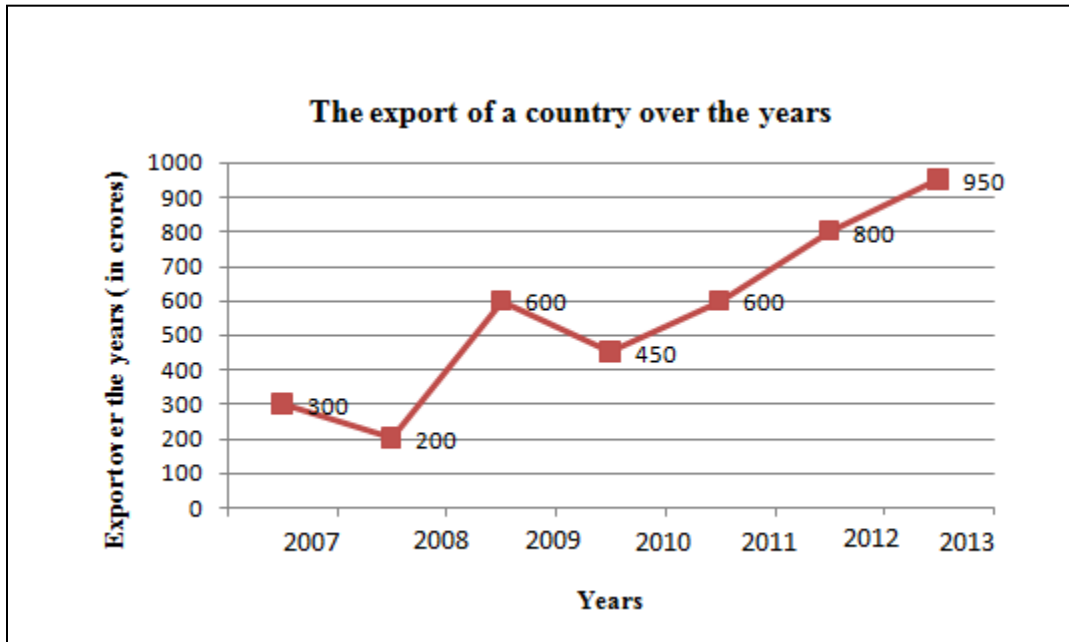
Percentage of females from various industries working in night shifts

Industries	Females
IT	20%
Gaming	20%
Call center	45%
Sales	60%
Banking	40%
Chemical industries	15%

16. What is the respective ratio of the men to the women working in night shifts from the call center industry?
 a) 9:11 b) 7:5 c) 11:9 d) 11:7
17. What is the total number of females working in night shifts from all the industries together?
 a) 14000 b) 25780 c) 13280 d) 15500
18. The number of women working in night shifts from the gaming industry is what percent of the total number of people working in the night shifts from all the industries together?
 a) 5.6% b) 3.6% c) 3.2% d) 3.7%
19. What is the difference between the total no of men & the total no of women working in night shifts from all the industries together?
 a) 26720 b) 35400 c) 30000 d) 13440

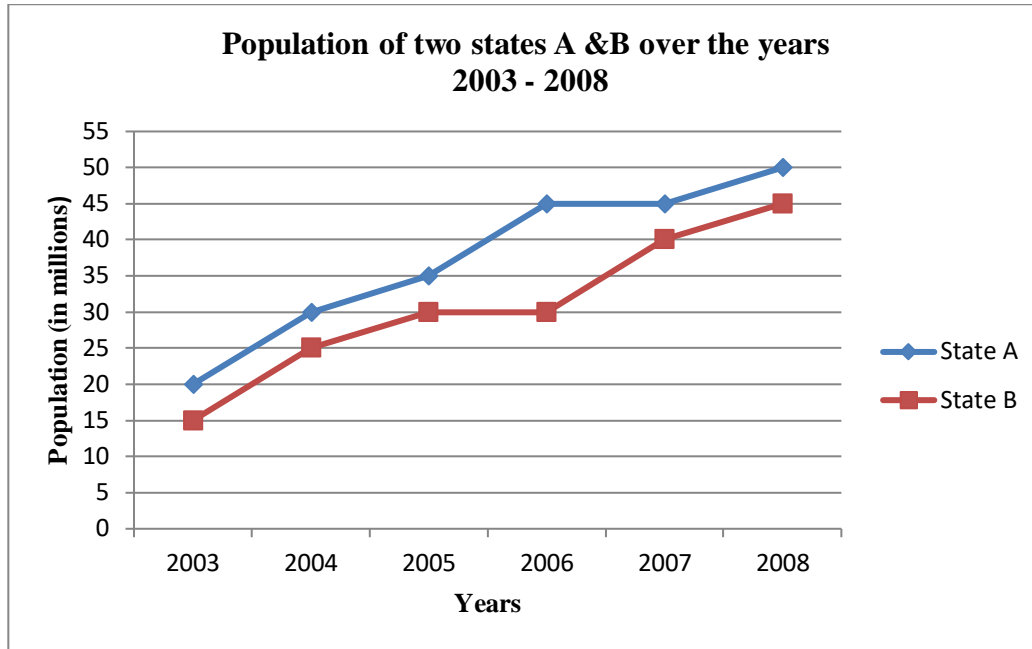
LINE CHARTS

- I. Directions (Q.No. 1 to 5): Study the following graph carefully and answer the given questions.



- Which year has the highest increase in exports as compared to the previous year?
 a) 2008 b) 2009 c) 2010 d) 2011
- What is the difference in exports in the years 2009 and 2010?
 a) 250 crore b) 200 crore c) 150 crore d) 100 crore
- What is the percent increase in exports from the lowest to the highest for the given years?
 a) 375% b) 250% c) 175% d) 125%
- Export in 2009 is approximately what percent of that of year 2010?
 a) 35% b) 66% c) 210% d) 133%
- What is the total exports (in crore) for the given years?
 a) 3900 crore b) 4150 crore c) 2750 crore d) 3750 crore

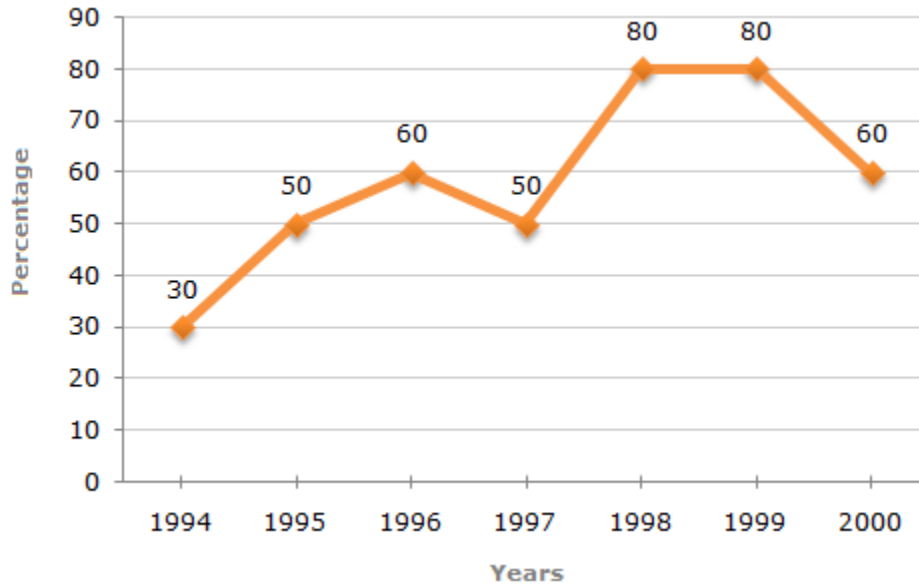
- II.** Directions (Q.No. 6 to 10): Study the following line chart carefully to answer the questions given below.



6. For state B, the rise in population from the previous year was the highest in which of the following years?
 a) 2008 b) 2006 c) 2005 d) 2004
7. What was the average population of state B(in millions) for the years together?
 a) 30.83 b) 35 c) 28.5 d) 26
8. What was the percent rise in population of state A in 2007 from the previous year?
 a) 25% b) $33\frac{1}{3}\%$ c) 33% d) 0%
9. What was the ratio of the total population of state A to that of state B for all the years together?
 a) 37:45 b) 37:43 c) 43:37 d) 45:37
10. Population of state A in 2005 was what percent of its total population for all the years together?
 a) $15\frac{5}{9}\%$ b) $17\frac{1}{9}\%$ c) $16\frac{2}{5}\%$ d) None of these

III. Directions (Q.No. 11 to 15): Study the line chart carefully and answer the questions given below.

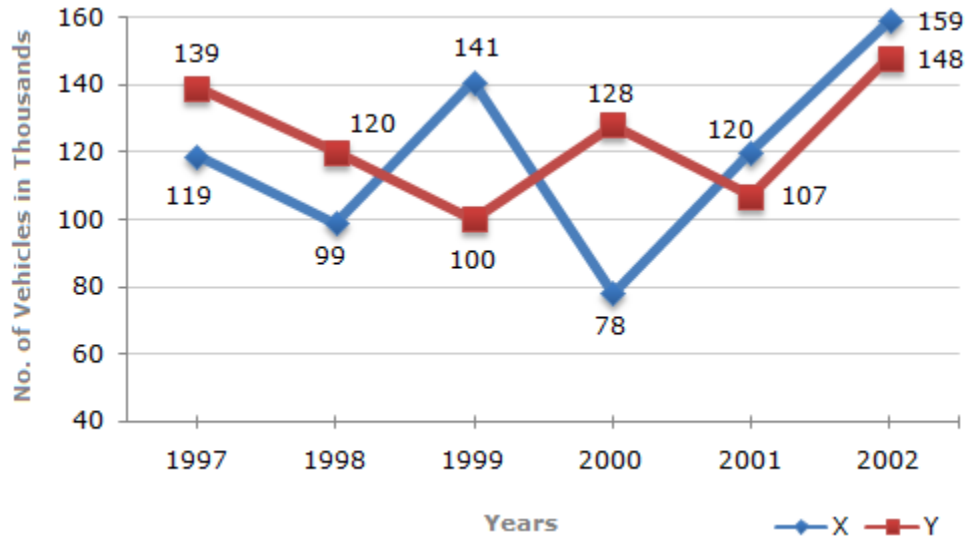
Percentage of Candidates passed in an Examination over the Years 1994-2000



11. The difference between the percentage of candidates passed was maximum in which of the following pairs of years?
 a) 1994 & 1995 b) 1997 & 1998 c) 1998 & 1999 d) 1999 & 2000
12. In which pair of years was the number of candidates passed the same?
 a) 1995 & 1997 b) 1995 & 2000 c) 1998 & 1999 d) Data inadequate
13. If the number of candidates passed in 1998 was 21200, what was the number of candidates appeared in 1998?
 a) 32000 b) 28500 c) 26500 d) 25000
14. If the total number of candidates appeared in 1996 and 1997 together was 47400, then the total number of candidates passed in these two years together was
 a) 34700 b) 32100 c) 31500 d) Data inadequate
15. The total number of candidates passed in 1999 and 2000 together was 33500 and the number of candidates appeared in 1999 was 26500. What was the number of candidates in 2000?
 a) 24500 b) 22000 c) 20500 d) 19000

- IV. Directions (Q.No. 16 to 19): Study the following line chart carefully to answer the questions given below.

Number of Vehicles Manufactured by Two companies over the Years (Number in Thousands)



16. What is the difference between the number of vehicles manufactured by Company Y in 2000 and 2001?
 a) 50000 b) 42000 c) 33000 d) 21000
17. What is the difference between the total productions of the two Companies in the given years?
 a) 19000 b) 22000 c) 26000 d) 28000
18. What is the average numbers of vehicles manufactured by Company X over the given period? (rounded off to nearest integer)
 a) 119333 b) 113666 c) 112778 d) 111223
19. In which of the following years, the difference between the productions of Companies X and Y was the maximum among the given years?
 a) 1997 b) 1998 c) 1999 d) 2000
20. The production of Company X in 1999 was what percent of the production of Company Y in the same year?
 a) 173% b) 164% c) 141% d) 97%

27. CLOCKS

Hands of a Clock

- ✓ Hour hand
- ✓ Minute hand
- ✓ Seconds hand

Hands and degree

In conventional clock, we have 12 hours format.

✓ Hour hand

The smaller or slower hand of a clock is called the hour hand. It makes two revolutions in a day.

Hour hand completes one rotation in 12 hours.

One rotation = 360 degree

i.e., 12 hours = 360 degree

∴ In 1 hour = 30 degree

In 1 minute = 0.5 degree

✓ Minute hand

The bigger or faster hand of a clock is called the minute hand. It makes one revolution in every hour.

Minute hand completes one rotation in 1 hour.

One rotation = 360 degree

i.e., 1 hour = 360 degree

∴ In 60 minutes = 360 degree

In 1 minute = 6 degree

Angle formed between hour hand and minute hand

$$✓ \quad A = 30H - \frac{11}{2}M \quad (\text{If minute hand is before hour hand})$$

$$✓ \quad -A = 30H - \frac{11}{2}M \quad (\text{If minute hand is after hour hand})$$

Where,

A → Angle formed between hour hand and minute hand

H → Hour

M → Minute

Gain concept

In 1 hour, minute hand covers 60 min spaces whereas the hour hand covers 5 min spaces. Therefore, minute hand gains $(60 - 5) = 55$ min in 1 hour.

- ✓ In 60 minutes → 55 minutes gain

Important concepts about hands and their position✓ **Hands of the clock coincide/together (0 degree)**

- The hands of the clock will be together once in one hour. But between 11`o clock to 1`o clock, they will be together only once at 12`o clock.
- In every 12 hours, hands of the clock coincide each other 11 times.
- In a day (24 hours), hands of the clock coincide each other 22 times.
- Hands of the clock coincide once in every $65\frac{5}{11}$ minutes.

$$A = 30 H - \frac{11}{2} M$$

$$0 = 30 (1) - \frac{11}{2} M$$

$$\frac{11}{2} M = 30$$

$$M = 30 \times \frac{2}{11}$$

$$M = \frac{60}{11}$$

$$M = 5\frac{5}{11} \text{ mins}$$

Hands of the clock coincide for every $60 + 5\frac{5}{11} = 65\frac{5}{11}$ mins.

✓ **Hands of the clock are opposite to each other (180 degree)**

- In every hour, the hands of the clock are at opposite direction only once. But between 5`o clock to 7`o clock, they will be in opposite direction only once at 6`o clock.
- In every 12 hours, hands of the clock are at opposite direction 11 times.
- In a day (24 hours), hands of the clock are at opposite direction 22 times.

✓ **Hands of the clock forms straight line**

When the angle between hour hand and minute hand is 0 degree and 180 degree, they form a straight line.

- 0 degree forms when the hands coincide each other.
- 180 degree forms when the hands are opposite to each other.

- In every 12 hours, hands of the clock form a straight line 11 times.
- In a day (24 hours), hands of the clock form a straight line 22 times.

✓ **Hands of the clock forms Right angle (90 degree)**

- The hands of the clock will be at right angle twice in one hour. But between 2`o clock to 4`o clock, they will be at right angle only three times.
- In every 12 hours, hands of the clock form a right angle 22 times.
- In a day (24 hours), hands of the clock form a right angle 44 times.

Reflex Angle

A reflex angle is an angle that is greater than 180 degrees but less than 360 degrees. A reflex angle always has a corresponding angle which lies on the other side of it.

$$\text{Reflex angle for } 90 \text{ degree} = 360 - 90 = 270 \text{ degree}$$

Mirror Image Timing

$$\text{Mirror image time} = 12 - \text{Actual time.}$$

DRILL

1. A clock is started at noon. By 10 minutes past 5, the hour hand has turned through how many degrees?
2. What is the angle between the minute hand and the hour hand of a clock when the time is 4:20?
3. At 3:40 the hour hand and the minute hand of a clock forms an angle of what degree?
4. At what time between 3'o clock and 4'o clock will both the needles coincide each other?
5. Find at what time between 8 and 9'o clock will the hands of a clock be in the same straight line but not together?
6. At what time between 4 and 5'o clock will the hands of a clock be at right angle?
7. At what time between 3'o clock and 4'o clock will the hands of a clock be 4 min apart?
8. How much does a watch gain per day if it hands coincide every 64 minutes?
9. A watch which gains uniformly is 5 min slow at 8'o clock in the morning on Sunday and it is 5 min 48 sec fast at 8 p.m. on the following Sunday. When did it show the correct time?
10. A clock gains 10 s in every 3 hours. If the clock was set right at 4:00 am on Monday morning, then what will be the time it indicate on Tuesday evening at 7:00 p.m?
11. A watch which he gains 5 seconds in 3 minutes was set right at 7 am. In the afternoon of the same day, when the watch indicated quarter past 4'o clock, what will be the true time?
12. If a clock shows 9:30, what time it will show in the mirror?
13. A clock strikes once at 1'o clock and twice at 2'o clock and so on. What is the total number of strikes in a day?
14. A clock strikes once at 1'o clock and twice at 2'o clock and so on. If it takes 10 sec to strike 10, what is the time will it take to strike 2?

PRACTISE QUESTIONS

1. An accurate clock shows 8'o clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2'o clock in the afternoon?
2. What is the reflex angle between the hands of a clock at 25 minutes past 10?
3. At what time between 2 and 3'o clock will the hands of a clock be together?
4. At what time between 4 and 5'o clock will the hands of a watch point in opposite direction?
5. At what time between 5:30 and 6 will the hands of a clock be at right angle?
6. At what point of time after 3'o clock hour hand and the minute hand are at right angle for the first time?
7. At what time between 5 and 6'o clock the hands of the clock are 3 minutes apart?
8. The minute hand of a clock over takes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?
9. A watch which gains uniformly is 2 minutes low at noon on Monday and is 4 min 48 sec fast at 2 pm on the following Monday. When did it show the correct time?
10. A clock is set right at 5 am. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 pm on the 3rd day?
11. Mirror shows 7:45, what is the actual time?
12. At the time between 7 and 8'o clock, correct to nearest minute, when will the hands of the clock form an angle of 84 degree?

28. CALENDAR

A calendar is chart or series of pages showing the days, weeks and months of a particular year. A calendar consists of 365 or 366 days divided into 12 months.

Ordinary Year

A year having 365 days is called an ordinary year
 ✓ 52 complete weeks + 1 extra day = 365 days

Leap Year

A leap year has 366 days (the extra day is 29th of February)
 ✓ 52 complete weeks + 2 extra days = 366 days.

To find leap year or not

A leap year is exactly divisible by 4 except for a century. For a century to be a leap year it must be divisible by 400. For example,

- ❖ Years like 1988, 2008 are leap year (divisible by 4).
- ❖ Centuries like 2000, 2400 are leap year (divisible by 400).
- ❖ Years like 1999, 2003 are not leap year (not divisible by 4).
- ❖ If Centuries like 1700, 1800 are not leap year (not divisible by 400).

Odd Days

Extra days, apart from the complete weeks in a given period are called odd days.
 ✓ An ordinary year has 1 odd day.
 ✓ A leap year has 2 odd days.

Odd days in a century

I. First century (1 to 100)

In first century, there are 76 Ordinary year and 24 Leap years.

Number of odd days in a century = $(76 \times 1 + 24 \times 2) = 124$

$$\Rightarrow \frac{124}{7} = 17 \text{ week} + 5 \text{ odd days}$$

∴ First century has 5 odd days.

II. Second century (1 to 200)

Up to second century, there are $76 \times 2 = 152$ Ordinary year and $24 \times 2 = 48$ Leap years.

Number of odd days in a century = $(152 \times 1 + 48 \times 2) = 248$ odd days

$$\Rightarrow \frac{248}{7} = 35 \text{ week} + 3 \text{ odd days}$$

∴ Second century has 3 odd days.

(or)

From 1 to 100 = 5 odd days

From 101 to 200 = 5 odd days

$$\Rightarrow 5 + 5 = 10 \text{ extra days}$$

$$\Rightarrow \frac{10}{7} = 1 \text{ week} + 3 \text{ odd days}$$

∴ Second century has 3 odd days

III. Third century (1 to 300)

Up to third century, there are $76 \times 3 = 228$ Ordinary year and $24 \times 3 = 72$ Leap years.

Number of odd days in a century = $(228 \times 1 + 72 \times 2) = 372$

$$\Rightarrow \frac{372}{7} = 53 \text{ week} + 1 \text{ odd days}$$

\therefore Third century has 1 odd day.

(or)

From 1 to 100 = 5 odd days

From 101 to 200 = 5 odd days

From 201 to 300 = 5 odd days

$$\Rightarrow 5 + 5 + 5 = 15 \text{ extra days}$$

$$\Rightarrow \frac{15}{7} = 2 \text{ week} + 1 \text{ odd days}$$

\therefore Third century has 1 odd day.

IV. Fourth century (1 to 400)

Up to fourth century, there are $(76 \times 4) - 1 = 303$ Ordinary year and $24 \times 3 = 97$ leap years.

(\because 400th year is a leap year)

From 301 to 400, there are 75 Ordinary year and 25 leap years.

Number of odd days in a century = $(76 \times 1 + 24 \times 2) = 124$

Number of odd days in a century = $(303 \times 1 + 97 \times 2) = 497$

$$= \frac{497}{7}$$

$$= 71 \text{ week} + 0 \text{ odd days}$$

\therefore Four century has 0 odd day.

(or)

From 1 to 100 = 5 odd days

From 101 to 200 = 5 odd days

From 201 to 300 = 5 odd days

From 301 to 400 = 6 odd days \because 400th year is a leap year

$$\Rightarrow 5 + 5 + 5 + 6 = 21 \text{ extra days}$$

$$\Rightarrow \frac{21}{7} = 3 \text{ week} + 0 \text{ odd days}$$

\therefore Fourth century has 0 odd day.

The last day of the century

- ✓ First century has 5 odd days. So it ends with **Friday**.
- ✓ Two centuries has 3 odd days. So it ends with **Wednesday**.
- ✓ Three century has 1 odd day. So it ends with **Monday**.
- ✓ Four centuries has 0 odd days. So it ends with **Sunday**.
- ✓ So the last day of a century cannot be **Tuesday, Thursday and Saturday**.

Month code

JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
	0 + 3	3 + 0	3 + 3	6 + 2 = 8 $\frac{8}{7} = 1$	1 + 3	4 + 2	6 + 3 = 9 $\frac{9}{7} = 2$	2 + 3	5 + 2 = 7 $\frac{7}{7} = 0$	0 + 3	3 + 2
0	3	3	6	1	4	6	2	5	0	3	5
$\frac{31}{7} = 3$	$\frac{28}{7} = 0$	$\frac{31}{7} = 3$	$\frac{30}{7} = 2$	$\frac{31}{7} = 3$	$\frac{30}{7} = 2$	$\frac{31}{7} = 3$	$\frac{31}{7} = 3$	$\frac{30}{7} = 2$	$\frac{31}{7} = 3$	$\frac{30}{7} = 2$	

Month code

January	0
February	3
March	3
April	6
May	1
June	4
July	6
August	2
September	5
October	0
November	3
December	5

Day code

Sunday	0
Monday	1
Tuesday	2
Wednesday	3
Thursday	4
Friday	5
Saturday	6

Year code

➤ **1600 – 1699 (100 years)**

In the given period, there are 75 Ordinary year and 25 Leap years.

(∵ 1600 is a leap year)

Number of odd days in a century = (75 x 1 + 25 x 2) = 125

$$\Rightarrow \frac{125}{7} = 17 \text{ week} + 6 \text{ odd days}$$

∴ From 1600 to 1699 there are 6 odd days.

Until 1699 there are 6 odd days.

➤ **1700 – 1799 (100 years)**

In the given period, there are 76 Ordinary year and 24 Leap years.

(∵ 1700 is not a leap year)

Number of odd days in a century = (76 x 1 + 24 x 2) = 124

$$\Rightarrow \frac{124}{7} = 17 \text{ week} + 5 \text{ odd days}$$

∴ From 1700 to 1799 there are 5 odd days.

Until 1799, there are 6 + 5 = 11 odd days.

$$\Rightarrow \frac{11}{7} = 1 \text{ week} + 4 \text{ odd days}$$

Until 1799, there are 4 odd days.

➤ **1800 – 1899 (100 years)**

In the given period, there are 76 Ordinary year and 24 Leap years.

(∵ 1800 is not a leap year)

Number of odd days in a century = $(76 \times 1 + 24 \times 2) = 124$

$$\Rightarrow \frac{124}{7} = 17 \text{ week} + 5 \text{ odd days}$$

∴ From 1800 to 1899 there are 5 odd days.

Until 1899, there are $4 + 5 = 9$ odd days.

$$\Rightarrow \frac{9}{7} = 1 \text{ week} + 2 \text{ odd days}$$

Until 1899, there are 2 odd days.

➤ **1900 – 1999 (100 years)**

In the given period, there are 76 Ordinary year and 24 Leap years.

(∵ 1900 is not a leap year)

Number of odd days in a century = $(76 \times 1 + 24 \times 2) = 124$

$$\Rightarrow \frac{124}{7} = 17 \text{ week} + 5 \text{ odd days}$$

∴ From 1800 to 1899 there are 5 odd days.

Until 1899, there are $2 + 5 = 7$ odd days.

$$\Rightarrow \frac{7}{7} = 1 \text{ week} + 0 \text{ odd days}$$

Until 1999, there are 0 odd days.

➤ **2000 – 2099 (100 years)**

In the given period, there are 75 Ordinary year and 25 Leap years.

(∵ 2000 is a leap year)

Same as 1600 to 1699

Until 2099, there are 0 odd days

Year code

1600 - 1699	6
1700 – 1799	4
1800 – 1899	2
1900 – 1999	0

The same repeats for every 400 centuries

2000 – 2099	6
2100 – 2199	4

Repetition of the calendars

- ✓ To find the repetition of the calendar, divide the given year by 4.
- ✓ Depending on the remainder value obtained, the repetition of calendar can be calculated as follows.

Remainder	Repeated after
0	28 years
1	6 years
2	11 years
3	11 years

Proof

Year	Odd days
2000	2
2001	1
2002	1
2003	1
2004	2
2005	1
2006	1
2007	1
2008	2
2009	1
2010	1
2011	1
2012	2
2013	1
2014	1
2015	1

Year	Odd days
2016	2
2017	1
2018	1
2019	1
2020	2
2021	1
2022	1
2023	1
2024	2
2025	1
2026	1
2027	1
2028	2
2029	1
2030	1
2031	1

i. 2001 calendar will be repeated in which year?

After 2001, 2007 begins with 0 odd days.

∴ 2001 calendar will repeats on 2007

Check:

$$\Rightarrow \frac{2001}{4} \Rightarrow \text{remainder is 1}$$

$$2001 + 6 = 2007$$

Hence proved.

ii. 2002 calendar will be repeated in which year?

After 2001, 2008 begins with 0 odd days

2002 is an ordinary year and 2008 is a leap year, so calendar repetition is not possible.

After 2008, 2013 begin with 0 odd days

∴ 2002 calendar will repeats on 2013

Check:

$$\Rightarrow \frac{2002}{4} \Rightarrow \text{remainder is 2}$$

$$2002 + 11 = 2013$$

Hence proved.

Repetition of the calendar among months of a year

To have same calendar, the month should begin with 0 odd days.

Month	Odd days
January	3
February	1 (leap year)
	0 (ordinary year)
March	3
April	2
May	3
June	2
July	3
August	3
September	2
October	3
November	2
December	3

Repetition applicable to ordinary years

- ✓ Calendar of January repeats on October.
- ✓ Calendar of February repeats on November.

Repetition applicable to leap years

- ✓ Calendar of January repeats on April and July.
- ✓ Calendar of February repeats on August.

Repetition applicable to both leap year and ordinary year

- ✓ Calendar of March repeats on November.
- ✓ Calendar of April repeats on July.
- ✓ Calendar of September repeats on December.

Day Gain/Loss concept✓ **Ordinary Year (± 1 day)**

- **If when we proceed forward by 1 yr, then 1 day is gained.**

For example, if 9th August 2013 is Friday, then 9th August 2014 has to be
Friday + 1 = Saturday.

- **If when we move backward by 1 yr, then 1 day is lost.**

For example, if 24th December 2013 is Tuesday, then 24th December 2012 has to be
Tuesday - 1 = Monday.

✓ **Leap Year (+ 2 days)**

- **When we proceed forward by 1 leap year, then 2 days are gained.**

For example, if it is Wednesday on 25th December 2011, then 25th December 2012 would be

$$\text{Wednesday} + 2 = \text{Friday} \quad \because 2012 \text{ is a leap year.}$$

- **When we move backward by 1 leap year, then 2 days are lost.**

For example, if it is Wednesday on 18th December 2012, then 18th December 2011 would be

$$\text{Wednesday} - 2 = \text{Monday} \quad \because 2012 \text{ is a leap year.}$$

✓ **Exception**

- **The day must have crossed 29th February for adding 2 days otherwise 1 day.**

Example 1: if 26th January 2011 is Wednesday, then 26th January 2012 would be

$$\text{Wednesday} + 1 = \text{Thursday}$$

(even if 2012 is leap year, we have added + 1 day because 29th February is not crossed).

Example 2: If 23rd March 2011 is Wednesday, then 23rd March 2012 would be

$$\text{Wednesday} + 2 = \text{Friday}$$

(+ 2 days 29th February of leap year is crossed).

To Find a Particular Day on the Basis of Given Day and Date

Step 1: Write the last two digit of the given year.

Step 2: Write the given date

Step 3: Write the quotient value obtained on dividing last two digit of the given year by 4.

Step 4: Write the corresponding month code.

Step 5: Write the corresponding year code.

Step 6: Add all the above data.

Step 7: Divide the obtained sum by 7.

Step 8: Find the day code for the obtained remainder value.

Note: If the given year is a leap year and the month is either January or February, then 1 should be subtracted from the remainder value obtained on step 7.

DRILL

1. What was the day of the week on 15th August 1947?
2. What was the day of the week on 26th January 1880?
3. On what dates of April 2001 did Wednesdays fall?
4. Today is Monday. What will be the day after 61 days?
5. Which of the given years is not a leap year?
a) 700 b) 800 c) 1200 d) 2000
6. How many days are there in X weeks X days?
7. The last day of the century year cannot be of what days?
8. Jan 1, 2007 was Monday, what day of the week was Jan 1, 2008?
9. If 8th Feb 2004 was Sunday. What day of the week was 8th Feb 2008?
10. On 6th March, 2005 Monday falls. What was the day of the week on 6th March 2004?
11. If Nov 5, 1987 was Wednesday. What day of the week was 4th April 1988?
12. What is the maximum gap between 2 successive leap years?
13. The calendar of the year 2024 can be used again in which year?
14. 2005 calendar will be repeated in which year?
15. If 2nd & 4th Saturdays and every Sunday is a holiday, then how many working days will be there in a month of 31 days beginning on a Friday?
16. How old are you if you are born in 1995?
17. The day before yesterday I was 25 years old, next year I will turn 28. How is it possible?
18. How many weekends are there in a year?
19. Which two months in a year have the same calendar?

PRACTISE QUESTIONS

1. What was the day of the week on 15th August 2010?
2. What was the day of the week on 16th July 1776?
3. What was the day of the week on 3rd March 2004?
4. On what dates of March 2005 did Thursdays fall?
5. Today is Friday. What will be the day after 42 days?
6. If 5th Jan 1991 was Saturday. What day was 4th March 1992?
7. If Sep 6, 1999 was Monday then what day was Oct 20, 1998?
8. If March 3, 2012 was Saturday, what day was June 23, 2010?
9. On March 6, 2007 Tuesday falls. What was the day of the week on 6th March 2004?
10. 2018 calendar will be repeated in which year?
11. The calendar of the year 1999 was repeated in which year?
12. If 2nd Saturdays and all Sundays are holidays in a 30 days month beginning on a Saturday, then how many working days are there in that month?

29. MENSURATION

Mensuration is the branch of mathematics that deals with the measurement of geometric figures and their parameters like length, volume, shape, surface area, lateral surface area, etc.

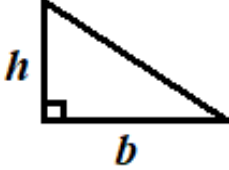
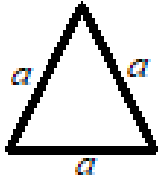
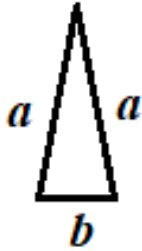
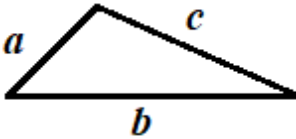
There are two types of Mensuration.

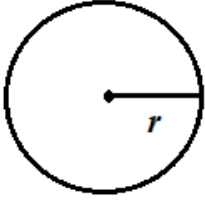
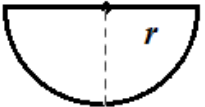
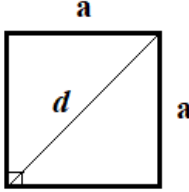
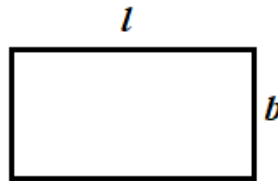
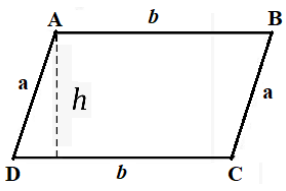
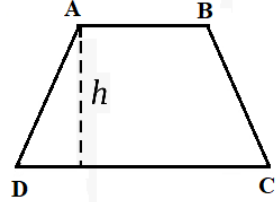
- ✓ 2 Dimensional shapes and
- ✓ 3 Dimensional shapes

2 Dimensional

A 2D diagram is a shape laid down on a plane by three or more straight lines or a closed segment. The 2D figures have length and width or breadth only. There is no height or depth.

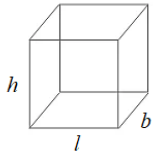
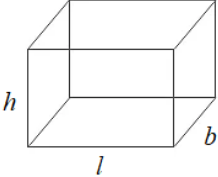
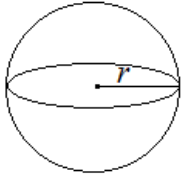

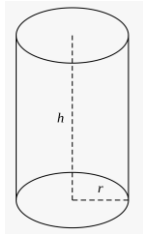
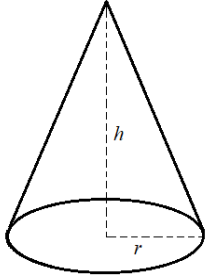
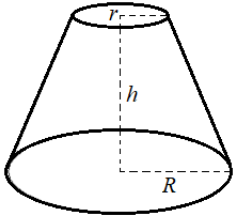
The parameters like Area (A) and Perimeter (P) can be measured in the 2 Dimensional shapes.

SHAPE	AREA (SQUARE UNITS)	PERIMETER (UNITS)	FIGURE
Right Angle Triangle	$\frac{1}{2} * b * h$	height + base + hypotenuse (hyp ² = b ² + h ²)	
Equilateral triangle Height, h = $\frac{\sqrt{3}}{2} * a$	$\frac{\sqrt{3}}{4} * a^2$	3a	
Isosceles Triangle Height, h = $\frac{1}{2} \sqrt{4a^2 - b^2}$	$\frac{1}{2} * b * h$	2a + b	
Scalene Triangle	$\frac{\sqrt{s(s-a)(s-b)(s-c)}}{1}$ Where, s = $\frac{a+b+c}{2}$	a + b + c	

<p>Circle (Diameter = 2r)</p>	πr^2	$2\pi r$ (or) πD	
<p>Semi - Circle</p>	$\frac{1}{2} \pi r^2$	$\pi r + 2\pi$	
<p>Square (Diagonal = $a\sqrt{2}$)</p>	a^2	$4a$	
<p>Rectangle (Diagonal = $\sqrt{l^2 + b^2}$)</p>	$l * b$	$2(l + b)$	
<p>Parallelogram</p>	$b * h$	$2(l + b)$	
<p>Trapezium</p>	$\frac{1}{2} h(a + c)$	$a + b + c + d$	

3 Dimensional

The 3D figures have length, breadth and height or depth. 3D shapes are made up of several 2D shapes. The parameters such as volume (V), Curved Surface Area (CSA), Lateral Surface Area (LSA) and Total Surface Area (TSA) can be measured in 3 Dimensional shapes.

SHAPE	Volume (Cubic units)	CSA or LSA (Square units)	Total Surface Area (TSA) (Square units)	FIGURE
Cube (Diagonal, $d = a\sqrt{3}$)	a^3	$LSA = 4 a^2$	$6 a^2$	
Cuboid (Diagonal, $d = \sqrt{l^2 + b^2 + h^2}$) Area of all 4 sides = $2 * h(l + b)$	$l * b * h$	$LSA = 2h(l + b)$	$2 (lb + bh + lh)$	
Sphere	$\frac{4}{3} \pi r^3$	$4 \pi r^2$	$4 \pi r^2$	
Hemisphere	$\frac{2}{3} \pi r^3$	$2 \pi r^2$	$3 \pi r^2$	
Cylinder	$\pi r^2 h$	$2 \pi r h$	$CSA + \text{Area of top and bottom circle}$ $= 2 \pi r h + 2 \pi r^2$ $= 2 \pi r (h + r)$	
Cone Slant height, $l = \sqrt{r^2 + h^2}$	$\frac{1}{3} \pi r^2 h$	$\pi r l$	$\pi r (r + l)$	
Frustum of the cone Slant height, $l = \sqrt{r^2 + (R - r)^2}$	$\frac{\pi h}{3} (r^2 + R^2 + rR)$	$\pi (R + r) l$	$\pi \{ (r + R) l + r^2 + R^2 \}$	

DRILL**2 DIMENSIONAL SHAPES****TRIANGLE**

1. Find the perimeter of a triangle with sides equal to 3 cm, 8 cm and 5 cm.
2. The perimeter of an equilateral triangle is 45 cm. Find its area.
3. A triangle with three equal sides has its area equal to $4\sqrt{3}$ sq.cm. What will be the perimeter of this triangle?
4. The perimeter of an isosceles triangle is 32 cm while its equal sides together measure 18cm. Find the third side and each of the equal sides.
5. The area of a right angled triangle is 42 sq cm. If its perpendicular is equal to 10 cm, find its base.
6. Find the area of a triangle whose sides are 26 cm, 28 cm and 30 cm.
7. If the area of circumference of an equilateral triangle is $\frac{25\pi}{3}$ cm², then find the radius of the inner circle of the triangle.
8. The base of a triangular wall is 7 times its height. If the cost of painting the wall at Rs.350 per 100 sq m is Rs.1225, then what is the base length?
9. The area of a right angled triangle is 24 cm and one of the sides containing the right angle is 6 cm. Find the altitude on the hypotenuse.
10. Three sides of a triangular field are of length 15 m, 20 m and 25 m long, respectively. Find the cost of sowing seeds in the field at the rate of Rs.5 per sq.m.
11. Two isosceles triangles have equal vertical angles and their corresponding sides are in the ratio of 3 : 7. What is the ratio of their areas?

CIRCLE

12. The inner circumference of a 7 m wide circular race track, is 220 m. Find the radius of the outer circle.
13. A wheel makes 2000 revolutions in covering a distance of 88 km. Find the radius of wheel.
14. If area of a square is 44 sq cm, find the area of the circle formed by the same perimeter.
15. If the area of a semi-circle be 77 sq m, find its perimeter.

16. A railing of 288 m is required for fencing a semi-circular park. Find the area of the park.
17. Find the area of a square inscribed in a circle of radius 5 cm.
18. The circumference of a circle is 50 cm. Find the side of the square inscribed in the circle.
19. The largest triangle is inscribed in a semi-circle of radius 7 cm. Find the area inside the semi-circle which is not occupied by triangle.
20. The inner circumference of a circular race track 7 m wide is 440 m. Find the radius of the outer circle.
21. Find the area of circle with maximum radius that can be inscribed in the rectangle of length 12 cm and breadth 8 cm.
22. Find the length of a rope by which a cow must be tethered in order that it may be able to graze an area of 154 sq m.

QUADRILATERAL

23. A square field has its area equal to 289 sq m. Find its side, perimeter and the diagonal of the square field.
24. The area of a rectangular field is 400 sq m. If the breadth of the field is 16 m, find the length of the field.
25. The length and breadth of a rectangle are 6 cm and 4 cm, respectively. What will be its diagonal?
26. The base of a parallelogram is twice its height. If the area of the parallelogram is 144 sq cm, find its height.
27. The difference between two parallel sides of a trapezium is 8 cm. The perpendicular distance between them is 38 cm. If the area of the trapezium is 950 cm², find the length of the parallel sides.
28. The ratio between the length and the breadth of a rectangle is 2:1. If breadth is 5 cm less than the length, what will be the perimeter of the rectangle?
29. The area of a rectangular field is 15 times the sum of its length and breadth. If the length of that field is 40 m, what is the breadth of that field?
30. A rectangular grassy plot 160 m x 45 m has a gravel path 3 m wide all the four sides inside it. Find the cost of gravelling the path at Rs.5 per sq m.
31. There is a garden of 140 m x 120 m and a gravel path is to be made of an equal width all around it, so as to take up just one-fourth of the garden. What must be breadth of path?

32. A rectangular grass plot 80 m x 60 m has two roads, each 10 m wide, running in the middle of it, one parallel to length and the other parallel to breadth. Find the area of the roads.
33. A ground 100 x 80 m has, two cross roads in its middle. The road parallel to the length is 5 m wide and the other road is 4 m wide, both roads are perpendicular to each other. Find the cost of laying the bricks at the rate of Rs.10 per m on the roads.

PROBLEMS ON INCREASE OR DECREASE ON SIZE

34. If the length and breadth of a rectangle are increased by 10% and 8%, respectively, then by what per cent will the area of that rectangle be increased?
35. If the length of a rectangle is increased by 5% and the breadth of the rectangle is decreased by 6%, then find the percentage change in area.
36. If sides of a square are increased by 5%, by what per cent, its area will be increased?
37. The radius of a circle is increased by 10%. Find the percentage increase in its area.

3 DIMENSIONAL SHAPES

CUBE

1. The diagonal of a cube is $12\sqrt{3}$ cm. Find its volume and surface area.
2. The surface area of a cube is 1350 sq cm. Find its volume.
3. Three cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find half of the surface area of the new cube.
4. How many cubes of 3 cm edge can be cut out of a cube of 18 cm edge?
5. The sides of three cubes of metal are 30 cm, 40 cm and 50 cm respectively. Find the side of new cube formed by melting these cubes together.
6. The volume of a cube is numerically equal to sum of its edges. What is the total surface area in square units?
7. When each side of a cube is increased by 20%, then find the increase in total surface area of a cube.

CUBOID

8. Find the volume and surface area of a cuboid 18 m long, 14 m broad and 7 m high.
9. What is the length of the largest pole that can be placed in a room 11 m long, 8 m broad and 9 m high?
10. A wooden box measures 10 cm x 6 cm x 5 cm. Thickness of wood is 2 cm. Find the volume of the wood required to make the box.
11. The length, breadth and height of a cuboid are in the ratio of 6:5:4 and its whole surface area is 66600 cm². What is its volume?
12. The area of 4 walls of a room is 520 sq m while its length and breadth are 15 m and 11 m respectively. Find the height of the room.
13. Find the maximum length of a pencil that can be kept in a rectangular box of dimensions 8 cm x 6 cm x 2 cm.
14. Internal length, breadth and height of a rectangular box are 10 cm, 8 cm and 6 cm, respectively. How many boxes are needed which can be packed in a cube whose volume is 6240 cu.cm?
15. A metal box measures 20 cm x 12 cm x 5 cm. Thickness of the metal is 1 cm. Find the volume of the metal required to make the box.
16. The whole surface area of a rectangular block is 8788 sq cm. If length, breadth and height are in the ratio of 4:3:2 then find the length.
17. The capacity of a cuboid tank of water is 50000 L. Find the breadth of the tank, if its length and depth are 2.5 m and 10 m, respectively.
18. The paint in certain container is sufficient to paint an area equal to 9.375 sq m. How many bricks of dimensions 22.5 cm x 10 cm x 7.5 cm can be painted out of this container?
19. What are the dimensions (length, breadth and height, respectively) of a cuboid with volume 720 cu cm, surface area 484 sq cm and the area of the base 72sqcm?
20. The floor of a rectangular hall has a perimeter 250 m. If the cost of painting the four walls at the rate of Rs.10 per sq m is Rs.15000, then find the height of the hall.

SPHERE AND HEMISPHERE

21. Find the volume and surface area of a sphere of diameter 14 cm.
22. Find volume, curved surface area and total surface area of a hemisphere of radius 7 cm.
23. A hemispherical bowl is made of steel and 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved area of the bowl.
24. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases.
25. What is the diameter of the largest circle lying on the surface of a sphere of surface area 616 sq cm?
26. The diameter of the Moon is approximately one-fourth of the diameter of the Earth. What is the ratio (approximate) of their volumes?
27. A sphere and a hemisphere have the same surface area. Find the ratio of their volume.
28. Find the number of lead balls of diameter 2 cm each that can be made from a sphere of diameter 16 cm.
29. A hemispherical bowl has 3.5 cm radius. It is to be painted inside as well as outside. Find the cost of painting it at the rate of Rs. 5 per 10 sq cm.
30. If the ratio of the diameters of two spheres is 3:5, then what is the ratio of their surface areas?
31. If 64 identical small spheres are made out of a big sphere of diameter 8 cm, what is surface area of each small sphere?
32. What will be the difference between total surface area and curved surface area of a hemisphere having 2 cm diameter?
33. A metallic sphere of radius 12 cm is melted into three smaller spheres. If the radii of two smaller spheres are 6 cm and 8 cm, find the radius of the third.
34. Weight of a solid metallic sphere of radius 4 cm is 4 kg. Find weight of a hollow sphere made with same metal, whose outer diameter is 16 cm and inner diameter is 12 cm.

CYLINDER AND HOLLOW CYLINDER

35. Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 14 cm and height 80 cm.
36. How many iron rods each of length 14 m and diameter 4 cm can be made out of 0.88 m³ of iron?
37. A hollow cylinder made of wood has thickness 1 cm while its external radius is 3 cm. If the height of the cylinder is 8 cm, then find the volume, curved surface area and total surface area of the cylinder.
38. If the lateral surface area of a cylinder is 94.2 sq cm and its height is 5 cm, then find the radius of its base. ($\pi = 3.14$)
39. A rod of 2 cm diameter and 30 cm length is converted into a wire of 3 m length of uniform thickness.
40. What is the height of a solid cylinder of radius 5 cm and total surface area is 660 sq. cm?
41. The ratio of the radii of two cylinders is 2:3 and the ratio of their heights is 5:3. Find the ratio of their volumes.
42. The curved surface area of a cylindrical pillar is 264 sq m and its volume is 924 m. Find the ratio of its diameter to its height.
43. If height of cylinder is decreased by 8%, while its radius remains unchanged, by what per cent does the volume decrease?
44. A drainage tile is a cylindrical shell 21 cm long. The inside and outside diameters are 4.5 cm and 5.1 cm, respectively. What is the volume of the clay required for the tile?
45. The diameter of a roller is 84 cm and its length 120 cm. It takes 500 complete revolutions to move once over to level a playground. Find the area of the playground (in sq m).
46. A cylindrical pillar is 50 cm in diameter and 3.5 m in height. Find the cost of painting the curved surface of the pillar at the rate of Rs. 10 per sq m.
47. A cylinder is surmounted by a cone at one end, a hemisphere at the other end. The common radius is 3.5 cm, the height of the cylinder is 6.5 cm and the total height of the structure is 12.8 cm. Find the volume of the structure.

CONE AND FRUSTUM OF CONE

48. The diameter of a right circular cone is 14 m and its slant height is 10 m. Find its curved surface area, total surface area and volume.
49. The diameter of a right circular cone is 14 m and its slant height is 10 m. Find the cost of painting the total surface of right circular cone if the rate of painting is 14 paise per sq m.
50. A frustum of a right circular cone has a diameter of base and top 20 cm and 12 cm, respectively and a height of 10 cm. Find the area of its whole surface and volume.
51. A cap is in the form of a right circular cone of base radius 7 cm and height 24 cm. Find the area of the sheet required to make 5 such caps.
52. If the volumes of two right circular cones are in the ratio 1:3 and their diameters are in the ratio 3:5, then find the ratio of their heights.
53. The diameters of two cones are equal. If their slant heights be in the ratio of 5:7, then find the ratio of their curved surface areas.
54. A conical cap has the base diameter 24 cm and height 16 cm. What is the cost of painting the surface of the cap at the rate of 70 paise per sq cm?
55. The curved surface area and the total surface area of a cylinder are in the ratio 1:2. If the total surface area of the right cylinder is 616 cm, then find its volume.

PRISM AND PYRAMID

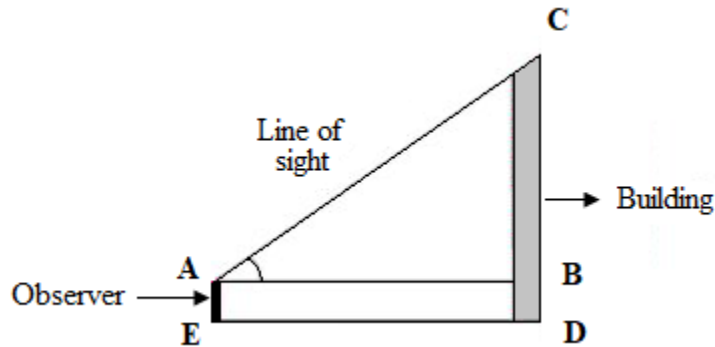
56. The base of a right prism is a square having side of 10 cm. If its height is 8 cm, then find the total surface area and volume of the prism.
57. The base of a pyramid is a square whose side is 10 cm. Its slant and vertical heights are 13 and 12 respectively. Then, find the total surface area and volume of the pyramid.
58. A prism has the base a right angled triangle whose sides adjacent to the right angle are 10 cm and 12 cm long. The height of the prism is 20 cm. The density of the material of the prism is 6 g/cu cm. Find the weight of the prism.
59. The perimeter of the triangular base of a right prism is 60 cm and the sides of the base are in the ratio 5:12:13. Then, find its volume (height of the prism being 50 cm).
60. A prism and a pyramid have the same base and the same height. Find the ratio of the volumes of the prism and the pyramid.

30. HEIGHT AND DISTANCE

It is an important application of trigonometry which helps us to find the height of any object and distance of that object from any point which are not directly measurable, if the angle of elevation/depression from a point is known.

Line of Sight

A line of sight is the line drawn from the eye of an observer to the point, where the object is viewed by the observer.

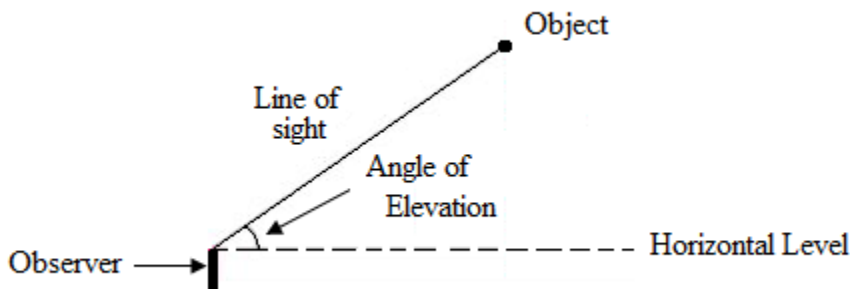


Horizontal Line

The line of sight which is parallel to ground level is known as horizontal line.

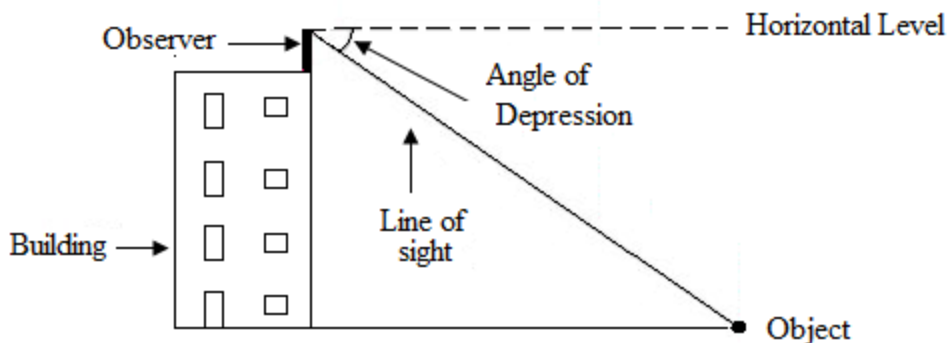
Angle of Elevation

The angle of elevation of the point viewed is the angle formed by the line of sight with the horizontal, when the point being viewed is above the horizontal level.



Angle of Depression

When the line of sight is below the horizontal level, the angle so formed by the line of sight with the horizontal is called the angle of depression.



Note:

- Angle of elevation and depression are always acute angles.
- Unless stated, it is assumed that the height of the observer is not considered.

DRILL

7. A tower stands vertically on the ground. From a point on the ground which is 30 m away from the foot of a tower, the angle of elevation of the top of the tower is found to be 45° . Find the height of the tower.
8. The shadow of a building standing on a level ground is found to be 40 m longer when the Sun's altitude becomes 30° from 45° . Find the height of the tower.
9. From a point A on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45° , respectively. If the bridge is at a height of 9 m from the surface of river, then find the width of the river.
10. A vertical post 15 ft high is broken at certain height and its upper part, not completely separated, meet the ground at an angle of 30° . Find the height at which the post is broken.
11. What is the angle of elevation of the Sun when the shadow of a pole is $\sqrt{3}$ times the length of the pole?
12. The shadow of a tower is 15 m, when the Sun's elevation is 30° . What is the length of the shadow, when the Sun's elevation is 60° ?
13. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30° . When it is 60° , then what is the height of the tower?
14. The angle of elevation of the top of an unfinished pillar at a point 150 m from its base is 30° . If the angle of elevation at the same point is to be 45° , then the pillar has to be raised to a height of how many metres?
15. From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower are observed to be 30° and 45° , respectively. What is the height of the tower?
16. A man standing at a point P is watching the top of elevation of 30° . The man walks some distance towards the tower and then his angle of elevation of the top of the tower is 60° . If the height of the tower is 30 m, then find the distance he moves.
17. The angles of elevation of the top of an inaccessible tower from two points on the same straight line from the base of the tower are 30° and 60° , respectively. If the points are separated at a distance of 100 m, then find the height of the tower.
18. Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?

19. If the angle of elevation of a tower from two distant points a and b ($a > b$) from its foot and in the same straight line and on the same side of it are 30° and 60° , then find the height of the tower.
20. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30° . If the distance between the two towers is n times the height of the shorter tower, then find the value of n?
21. A man standing in one corner of a square football field observes that the angle subtended by a pole in the corner just diagonally opposite to this corner is 60° . When he retires 80 m from the corner, along the same straight line, he finds the angle to be 30° . Find the length of the field.

PRACTISE QUESTIONS

11. What is the angle of elevation of the Sun, when the shadow of a pole of height x meter is $\frac{x}{\sqrt{3}}$ m?
12. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts a shadow of 40 m long on the ground. Find the height of the tower.
13. The tops of two poles of height 24 m and 36 m are connected by a wire. If the wire makes an angle of 60° with the horizontal, then find the length of the wire.
14. The angles of depression of two ships from the top of a lighthouse are 45° and 30° . If the ships are 120 m apart, then find the height of the lighthouse.
15. The angle of elevation of the tip of a tower from a point on the ground is 45° . Moving 21m directly towards the base of the tower, the angle of elevation changes to 60° . Find the height of the tower (approximately)?
16. On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30° to 45° . Find the height of the chimney.
17. The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building, if the height of the tower is 75 m and the angle of elevation of the top of the tower from the bottom of the building is 60° ?
18. The angles of elevation of the top of a tower from two points which are at distances of 10m and 5m from the base of the tower and in the same straight line with it are complementary. Find the height of the tower.
19. At the foot of a mountain, the elevation of its summit is 45° . After ascending 2 km towards the mountain upon an incline of 30° , the elevation changes to 60° . Find the height of the mountain.
20. A spherical balloon of radius r subtends angle 60° at the eye of an observer. If the angle of elevation of its centre is 60° and h is the height of the centre of the balloon, then which one of the following is correct?
 a) $h = r$ b) $h = \sqrt{2} r$ c) $h = \sqrt{3} r$ d) $h = 2r$

SECTION – II

**LOGICAL
REASONING**

1. CODING AND DECODING

TYPE 1

Example

In certain code, BOXER is written as AQWGQ. How VISIT is written in that code?

- a) UKRKU b) UKRKS c) WKRKU d) WKRKS

Solution:

Here, letter at the odd places have been written one letter back (-1). Letters at the even places have been written two letters ahead (+2) in the coded word, as their positions in the alphabet.

Similarly, VISIT is written as UKRKS.

DRILL

- If FESTIVAL is coded as MBWJUTFG then OPIUM would be coded as
a) NOHTL b) NTHNO c) NVJQP d) MUIPO
- If SYMBOL is coded as NZTMPC then NUMBER would be coded as
a) NVOSEC b) NVSSFC c) NUOSFC d) NVOSEC
- In a code Language if POSE is coded as OQNPRTDF, then TYPE will be coded as
a) SUZXQOQFD b) SUXZOQDF c) USZXQOQFD d) USXZOQDF

TYPE 2

Example

In a certain code PLANT is written as \$@2*© and YIELD is written as β64@% how is DELAY written in that code?

- a) β4*2% b) β4@2% c) %42@β d) %4@2β

Solution:

PLANT is coded as

$$P \rightarrow \$, L \rightarrow @, A \rightarrow 2, N \rightarrow *, T \rightarrow \text{©} \quad \dots(i)$$

and YIELD is coded as

$$Y \rightarrow \beta, I \rightarrow 6, E \rightarrow 4, L \rightarrow @, D \rightarrow \% \quad \dots(ii)$$

From Eqs (i) and (ii), the code for DONE is written as

$$\begin{array}{c} D E L A Y \\ \downarrow \downarrow \downarrow \downarrow \downarrow \\ \% 4 @ 2 \beta \end{array}$$

DRILL

- If ROSE is coded as 6821, CHAIR is coded as 73456 and PREACH is coded as 961473, what will be the code for SEARCH?
a) 246173 b) 214673 c) 214763 d) 216473
- In a certain code, RAID is written as %#*\$, RIPE is written as % * @ ©. How is DEAR written up in that code?
a) \$ * 37 b) 3\$7* c) \$©#* d) \$©#%
- In a certain code, ROPE is written as %57\$, DOUBT is written as 35#8* and LIVE is written as @24\$. How is TROUBLE written in that code?
a) *%5#8@\$ b) *%#58@\$ c) *%5#8@4 d) *%#58\$@

TYPE 3**Example**

If white is called blue, blue is called red, red is called yellow, yellow is called green, green is called black, black is called violet and violet is called orange, what would be the colour of human blood?

- a) Red b) Green c) Yellow d) Violet

Solution:

The colour of the human blood is 'red'. As per the given data, 'red' is called 'yellow'. So, the colour of human blood is 'yellow'.

DRILL

- If the animals which can walk are called 'swimmers', animals which can crawl are called 'flying', those living in water are called 'snakes' and those which fly in the sky are called 'hunters', what will be a lizard called?
a) Swimmers b) Snakes c) Flying d) Hunters
- If wall is called window, window is called door, door is called floor, floor is called roof and roof is called ventilator, what will a person stand on?
a) Window b) Wall c) Floor d) Roof
- If air is called green, green is called blue, blue is called sky, sky is called yellow, yellow is called water and water is called pink, then what is the colour of clear sky?
a) Blue b) Sky c) Yellow d) Water
- If sand is called air, air is called plateau, plateau is called well, well is called island and island is called sky, then from where will a woman draw water?
a) Well b) Island c) Sky d) Air

TYPE 4

Example

Direction (Q.No.1 to 5): Study the following information carefully and answer the questions accordingly.

In a certain code language,

‘smart work earn reward’ is written as ‘**ab ir ha ig**’

‘earn get reward to fame’ is written as ‘**af ig am ir sr**’

‘salary to get work now’ is written as ‘**am oj on ab af**’

‘to better life earn’ is written as ‘**ik ir ot af**’

- What may be the possible code for ‘better life quickly’ in the given code language?
a) ik ot on b) ot oj ik c) oj on ik d) ik ot ij e) ik ig ot
- What is the code for the word ‘salary’ in the language?
a) oj b) on c) Either ‘oj’ or ‘on’ d) Either ‘ik’ or ‘ot’ e) am
- What is the code for the word ‘better’ in the given language?
a) ik b) ot c) Either ‘oj’ or ‘on’ d) ir e) Either ‘ik’ or ‘ot’
- In the given code language what is the code for the word ‘smart’?
a) ab b) ir c) ha d) ig e) ik
- In the given code language, the code ‘sr’ stands for which word?
a) earn b) reward c) to d) fame e) life

Solution:

‘ smart **work** **earn** **reward** ’ is written as ‘ **ab** **ir** **ha** **ig** ’
 ‘ **earn** **get** **reward** **to** fame ’ is written as ‘ **af** **ig** **am** **ir** **sr** ’
 ‘ salary **to** **get** **work** now ’ is written as ‘ **am** **oj** **on** **ab** **af** ’
 ‘ **to** better life **earn** ’ is written as ‘ **ik** **ir** **ot** **af** ’

- ⇒ Smart is coded as **ha**
- ⇒ Work is coded as **ab**
- ⇒ Earn is coded as **ir**
- ⇒ Reward is coded as **ig**
- ⇒ Get is coded as **am**
- ⇒ To is coded as **af**
- ⇒ Fame is coded as **sr**
- ⇒ Salary is coded as **oj (or) on**
- ⇒ Now is coded as **oj (or) on**
- ⇒ Better is coded as **ik (or) ot**
- ⇒ Life is coded as **ik (or) ot**

DRILL

Direction (Q.No.1 to 5): Study the following information carefully and answer the questions accordingly.

In a certain code language,

‘she who knows vikram’ is written as ‘**am oc eh xm**’

‘vikram was a bad driver’ is written as ‘**xm hm al as xo**’

‘karan knows vikram’ is written as ‘**xm eh lk**’

‘who was driver of karan’ is written as ‘**lk hm oc ez xo**’

(All the codes are in two letter format)

1. What does the code ‘al’ stand for in the given code language?
a) vikram b) was c) a d) bad e) Either ‘a’ or ‘bad’
2. What is the code for ‘driver’?
a) lk b) xo c) hm d) ez e) Either ‘xo’ or ‘hm’
3. In the given code language, which of the following means ‘a bad driver’?
a) al as hm b) as al xo c) xo as hm d) Either (a) or (b) e) xm hm al
4. What does the code ‘oc’ stand for?
a) who b) knows c) she d) vikram e) Either (a) or (c)
5. What is the code for ‘she’ in the given code language?
a) am b) eh c) oc d) xm e) hm

TYPE 5**Example**

Direction (Q.No.1 to 5): Study the following information carefully and answer the questions accordingly.

‘given time simple plan’ is written as ‘**@E4 &N4 %N5 #E6**’

‘tired solution plant great’ is written as ‘**#N8 @D5 %T5 &T5**’

‘sick point good turn’ is written as ‘**#K4 %D4 @N4 &T5**’

‘garden sister phone team’ is written as ‘**&E5 #R6 %N6 @M4**’

1. Which of the following code for ‘translate’?
a) @E8 b) @E9 c) #E8 d) #T8 e) #T9

2. Which of the following code for 'plan'?
- a) %N5 b) @E4 c) &N4 d) #E6 e) None of these
3. In the given code language, what does the code '%D4'?
- a) point b) turn c) sick d) good e) None of these

Solution:**Logic to decode this code is**

- As we all know there is always a concept behind this which we need to find out. Looking at the questions, we can see that in all the steps, all 4 words start with the same alphabet. They all start with- G, T, S, and P, and their code word also start with the same symbol in each step- @, %, &, and #. So we just have to find which of the first alphabet represents the particular symbol.
- Now we can see that their code words end with digits. When we look at those digits, they are the total number of letters for each word.
- Now we can see the middle codeword is the last letter of that particular word.

DRILL

Direction (Q.No.1 to 5): Study the following information carefully and answer the questions accordingly.

In a certain coded language,

“Fresh Mind Happy Life” is coded as “H#14 M@17 K#33 U@17”

“Kashi City Of Temple” is coded as “S#20 G@28 L@21 O@25”

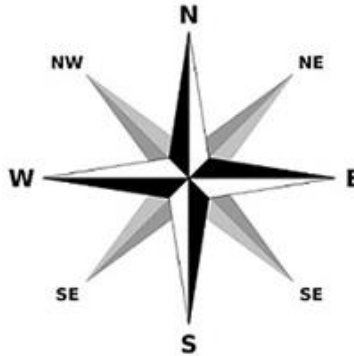
“Sarnath Belongs To Varanasi” is coded as “G#27 T#21 G@35 H@31”

“One Airport In Banaras” is coded as “M#20 I#21 R@23 Z#21”

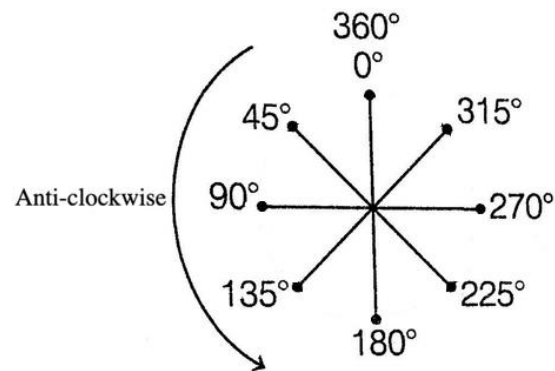
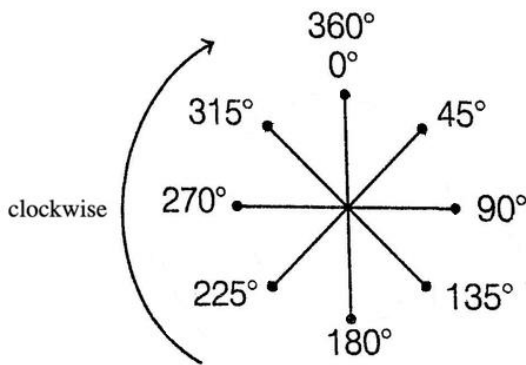
1. What is the code for 'excellent'?
- a) O@25 b) O#25 c) M@25 d) M#25
2. What is the code for 'collect'?
- a) X@23 b) Z@23 c) X#25 d) X#23
3. What is the code for 'life'?
- a) I@17 b) U@17 c) H#14 d) M@17
4. What is the code for 'success'?
- a) H#38 b) S#38 c) F@14 d) H#28
5. What is the code for 'smart'?
- a) I#29 b) R#39 c) I#39 d) M#19

2. DIRECTIONS SENSE

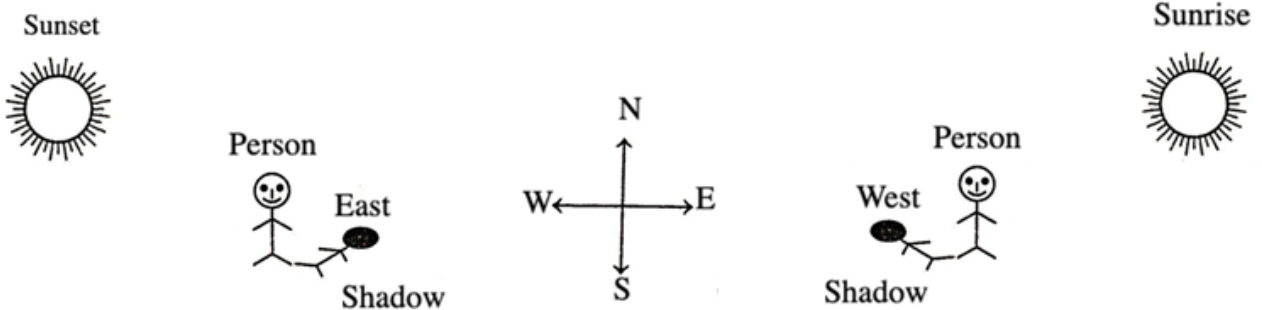
There are four major directions namely **East (E)**, **West (W)**, **North (N)** and **South (S)** and four intermediate directions, which are **North-East (NE)**, **North-West (NW)**, **South-East (SE)**, and **South-West (SW)**.



To solve the direction sense questions, we need to make a sketch for the provided data and according to that sketch we can easily get the answer.



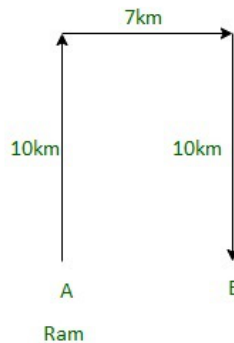
Some facts about shadow formed due to sunlight:



- ✓ In the morning, as the sun rises from the east, the shadow of any objects formed is in the west direction.
- ✓ At noon, or 12 pm, when the sun is exactly on top, no shadow will form.
- ✓ In the evening, as the sun sets in the West, so the shadow of any objects formed is in the East.

Example 1:

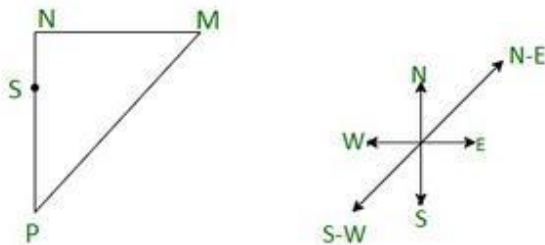
Ram starts from a point A walks 10 km north, then turns right and walks for 7 km, then turns right again and walks for another 10 km and reaches point B. How far is Ram from the starting point?

Solution:

So, from the above image it is clear that ram is 7 km away from the starting point.

Example 2:

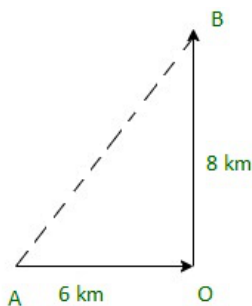
M is in the East of N, which is in the North of S. If P is in South of S, then in which direction of M is P?

Solution:

So, P is south-west of M

Example 3:

Sam started walking from point A towards East and walked for 6 km and then he turned to the left and walked for 8 km to reach point B. How far was he from the starting point?

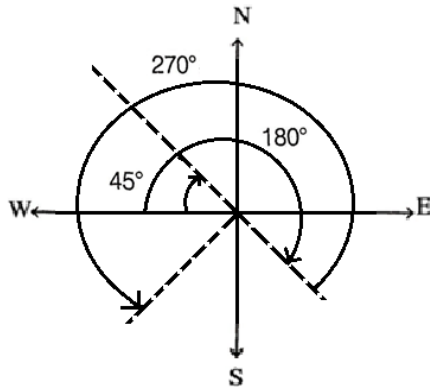
Solution:

Now, the distance between A and B = $\sqrt{6^2 + 8^2}$ (By Pythagoras' theorem)
 $= \sqrt{100}$
 $= 10 \text{ km}$

Therefore, the distance from the starting point is 10 km.

Example 4:

A man is facing west. He turns 45 degree in the clockwise direction and then another 180 degree in the same direction and then 270 degree in the anticlockwise direction. Find which direction is he facing now?

Solution:

Hence, he is facing south-west direction.

Example 5:

One morning after sunrise, Mahesh was standing facing a pole. The shadow of the pole fell exactly to his right. To which direction was he facing?

Solution:

The sun rises in the East (E) in the morning.

As the shadow of Mahesh falls to his right, he must be facing South.

Hence, the answer is the south.

DRILL

1. Rohit walked 25 metres towards South. Then he turned to his left and walked 20 metres. He then turned to his left and walked 25 metres. He again turned to his right and walked 15 metres. At what distance & direction is he from the starting point?
2. I am facing south. I turn right and walk 20 metres. Then I turn right again and walk 10 metres. Then I turn left and walk 10 metres and then turning right walk 20 metres. Then I turn right again and walk 60 metres. In which direction am I standing from starting point?
3. A man is facing north. He turns 45 degree in the clockwise direction and then another 180 degree in the same direction and then 45 degree in the anticlockwise direction. Find in which direction is he facing now?
4. A child is looking for his father. He went 90 meters in the east before turning to his right. He went 20 meters before turning to his right again to look for his father at his uncle's place 30 meters from this point. His father was not there. From there he went 100 meters to his north before meeting his father in a street. How far did the son meet his father from starting point?

5. A man leaves for his office from his house. He walks towards east. After moving a distance of 20m, he turns south and walks 10 m. Then he walks 35 m towards west and further 5 m towards north. He then turns towards east and walks 15m. What is the straight distance between his initial and final position?
6. One morning Udai and Vishal were talking to each other face to face at a crossing. If Vishal's shadow was exactly to the left of Udai, which direction was Udai facing?
7. One morning after sunrise Senthil while going to school met Karthi at road crossing. Karthi's shadow was exactly to the right of Senthil. If they were face to face which direction was Senthil facing?
8. Rahul put his watch on the table in such a way that at 6 pm hour hands points to north. In which direction the minute hand will point at 9:15 pm.

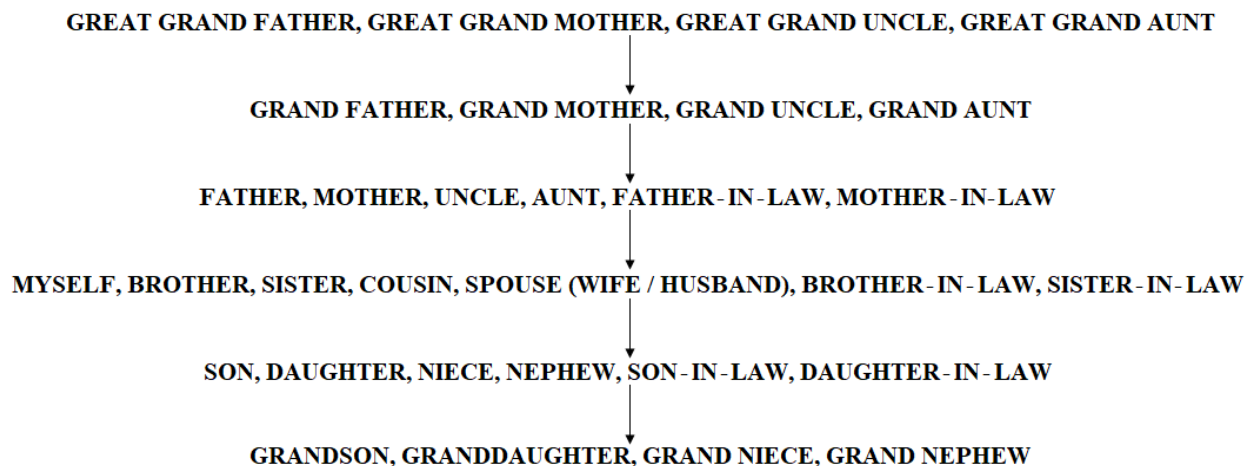
PRACTICE QUESTIONS:

1. One day Raviraj left home and cycled 20 Km southwards. He turned right and cycled 10 km and turned right again and cycled 20 Km and turned left and cycled 20 Km. How many kilometres will he have to cycle to reach his home straight?
2. Lokesh went 15 km to the North from his house. Then he turned west and covered 10 km. Then he turned south and covered 5 km. Finally turning to east he covered 10 km. In which direction is he now from his house?
3. A man is facing west. He turns 45 degree in the clockwise direction and then another 180 degree in the same direction and then 270 degree in the anticlockwise direction. Find in which direction is he facing now.
4. Kunal walks 10 km towards north. From there he walks 6 Km towards south. Then he walks 3 Km towards east. How far and in which direction is he with reference to his starting point?
5. A man is facing north-west. He turns 90° in the clockwise direction and then 135° in the anticlockwise direction. Which direction is he facing now?
6. A man is facing south. He turns 135° anti-clockwise direction and then 180° clockwise direction. Which direction is he facing now?
7. In the morning X and Y are walking towards each other in a park. When they meet each other, Y's shadow falls straight in front of X. In which direction was X facing?
8. In a compass, west direction is shown as south. As per the compass, which direction should a man go to, if he wishes to move towards the east?

3. BLOOD RELATIONS

The blood relationship is about the relationship of a particular person with another person on the same family based on the chain of relationships between other members of that family. To solve these types of problems, the candidate should have the knowledge of family relationships in right earnest and he has to interpret the vital link that would provide a clue to the relation of two particular persons as asked in the question. With that knowledge, candidate should sketch the situation by making a diagram based on the statements given in the direction part of the question. This would make the chain of relationship clear for conclusions.

Family Tree

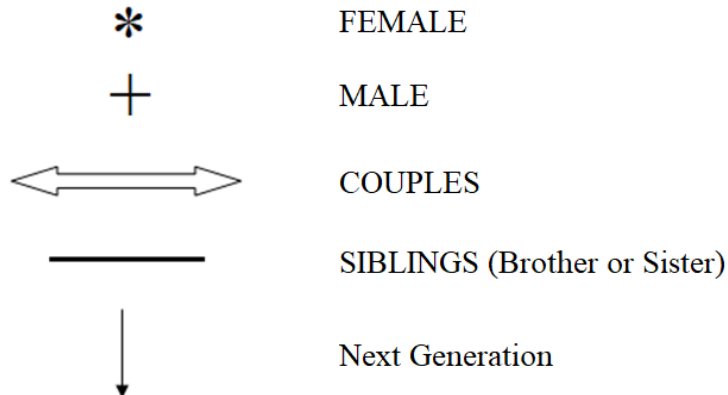


Basic Relations

- Mother's or Father's Father - **Grand Father**
- Mother's or Father's Mother - **Grand Mother**
- Grand Father's or Grand Mother's Brother - **Grand Uncle**
- Grand Father's or Grand Mother's Sister - **Grand Aunt**
- Mother's or Father's Brother - **Uncle**
- Mother's or Father's Sister - **Aunt**
- Children's Children - **Grand Children** (Grand Son, Grand Daughter)
- Uncle or Aunt's Son / Daughter - **Cousin**
- Son's Wife - **Daughter-in-law**
- Daughter Husband - **Son-in-law**
- Sister's or Brother's Son - **Nephew**
- Sister's or Brother's Daughter - **Niece**
- Husband's or Wife's Brother - **Brother-in-law**
- Sister's Husband - **Brother-in-law**
- Husband's or Wife's Sister or Brother Wife - **Sister-in-law**
- Brother Wife - **Sister-in-law**
- Grand Father's or Grand Mother's Son - **Father or Uncle**
- Grand Father's or Grand Mother's Daughter - **Mother or Aunt**

Important Terms

- ✓ Children of the same parent - **Siblings**
- ✓ Father side relations - **Paternal relatives**
- ✓ Mother side relations - **Maternal relatives**
- ✓ One's husband/wife - **Spouse**

Symbolic Representation**TYPE I**

1. A is the mother of B. C is the son of A. D is the brother of E. E is the daughter of B. How is D related to A?
2. Deepak is brother of Ravi. Renu is sister of Arun. Ravi is son of Renu. How is Deepak related to Renu?
3. P is the brother of D. X is the sister of P. A is the brother of F. F is the daughter of D. M is the father of X. Who is the uncle of A?
4. A is the father of C and D is the son of B. E is the brother of A. If C is the sister of D, then how is B related to E?
5. Q's mother is sister of P and daughter of M. S is daughter of P and sister of T. How is M related to T?

TYPE II

1. There are six persons A, B, C, D, E and F. C is the sister of F. B is the brother of E's husband. D is father of A and grandfather of F. A & B are brothers. There are two fathers, three brothers and a mother in the group.
 - i) Who is the mother?
 - ii) Who is E's husband?
 - iii) How many male members are there in the group?
 - iv) Mention the group of brother?
 - v) How is F related to E?

2. A is the father of C. But C is not his son. E is the daughter of C. F is the spouse of A. B is the brother of C. D is the son of B. G is the spouse of B. H is the father of G.
- Who is the grandmother of D?
 - Who is the son of F?
 - How is G related to C?
 - How is D related to E?
 - Who is the aunt of D?
3. A family consists of six members P, Q, R, X, Y and Z. Q is the son of R but R is not the mother of Q. P and R are a married couple. Y is the brother of R. X is the daughter of P. Z is the brother of P.
- Who is the brother-in-law of R?
 - Who is the father of Q?
 - How many children does P have?
 - How many female members are there in the family?
 - How is Q related to X?
 - Which is the pair of brothers?
 - P & X
 - P & Z
 - Q & X
 - R & Y

TYPE III

Directions (1-4)

A – B means A is daughter of B

A + B means A is wife of B

A ÷ B means A is father of B

A × B means A is son of B

- In the expression $P \times R - S$, how P is related to S?
- Which of the following means S is son-in-law of P?
 - $P + R \times S$
 - $P \div R \times S$
 - $P + R \div S$
 - $P \div R + S$
- In the expression $P - Q + S$, how is S related to P?
- How P is related to S in the expression $P \times Q \div S$?

TYPE IV

- Pointing to a photograph Vimal said, “She is the daughter of my grandfather’s only son”. How is Vimal related to the girl in the photo?
- Pointing to a photograph a man said “She is the daughter of the only son of my father’s wife” How is the girl in photograph related to the man?
- Pointing to a man a woman said, “His mother is the only daughter of my mother”. How is the woman related to the man?
- Pointing to a girl in the photograph Amar said, “Her mother’s brother is the only son of my mother’s father”. How is the girl’s mother related to Amar?
 - Mother
 - Aunt
 - Either A or B
 - Can’t be determined

4. SEATING ARRANGEMENT

The seating arrangement is the arrangement of people/objects logically. There are different types of seating arrangement as follows.

- ✓ Linear Arrangement
- ✓ Square/Rectangular Arrangement
- ✓ Circular Arrangement
- ✓ Triangular arrangement
- ✓ Hexagonal arrangement
- ✓ Pentagon arrangement

Basic Techniques

- This concept involves the arrangement of people in many possible ways.
- While solving questions, first start with the clues with more connectors.
- In these types of questions, we have to arrange a group of persons as to satisfying certain conditions.
- In any seating arrangement problem, the easiest way is eliminate one by one from the possible cases using the given statements.

Rules for seating arrangement

- ✓ Identifying the right and left in a given seating arrangement.
- ✓ We cannot assume left as an immediate left, if it is not mentioned in the question.
- ✓ In the case of a circular arrangement, if nothing is mention regarding the direction they are facing then by default take it as facing Centre.
- ✓ In the case of a linear arrangement, if nothing is mention regarding the direction they are facing then by default take it as facing North.

Example 1:

B, L, M, N, P, and Q are in a row. P and Q are in the center, B and L are at the ends. M is sitting to the left of B. Who is to the right of L?

Solution:

The seating arrangement is as follows.

L—N—P—Q—M—B

Therefore, the right of L is N.

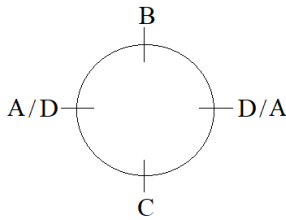
Example 2:

Four girls A, B, C and D are sitting around a circle facing the centre. B and C are in front of each other, which of the following is definitely true?

- a) A is to the left of C
- b) A is not between B and C
- c) D is to the left of C
- d) A and D are in front of each other

Solution:

The arrangement is as follows.



Therefore, the right of L is N.

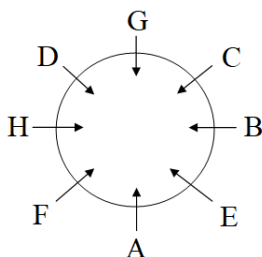
Example 3:

Eight friends, A, B, C, D, E, F, G and H are sitting in a circle facing the centre, not necessarily in the same order with conditions like,

- D sits third to the left of A.
- E sits to the immediate right of A.
- B is third to the left of D.
- G is second to the right of B.
- C is an immediate neighbour of B.
- C is third to the left of H.

Three of the following four are alike in a certain way based on the information given above and so form a group. Which is the one that does not belong to that group?

- a) DC
- b) AH
- c) EF
- d) CB

Solution: d) CB

CB is the odd one among the given options, since CB are neighbours

DRILL

1. P, Q, R, S, T, U, V and W are sitting around the circle and are facing centre. P is second to the right of T who is the neighbour of R and V. S is not the neighbour of P & Q. V is the neighbour of U.
- i. Which of the following are not neighbours?

a) RV	b) UV	c) RP	d) QW
-------	-------	-------	-------
 - ii. Which one is immediate right to the V?

a) P	b) U	c) R	d) T
------	------	------	------
 - iii. Which of the following is correct?

a) P is to the immediate right of Q	b) R is between U and V
c) Q is to the immediate left of W	d) U is between W and S
 - iv. What is the position of S?

a) Between U and V	b) Second to the right of P
c) To the immediate right of W	d) Data inadequate
2. Six friends are sitting in a circular table. Deepa is between Prakash and Pankaj. Priti is between Mukesh and Lalit. Prakash and Mukesh are opposite to each other.
- i. Who is sitting right to Prakash?

a) Mukesh	b) Deepa	c) Lalit	d) Either (b) or (c)
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 - ii. Who is just right to Pankaj?

a) Deepa	b) Lalit	c) Mukesh	d) Either Deepa or Mukesh
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 - iii. Who are the neighbours of Mukesh?

a) Prakash and Deepa	b) Deepa and Priti
c) Priti and Pankaj	d) Lalit and Priti
 - iv. Who is sitting opposite to Priti?

a) Prakash	b) Deepa	c) Pankaj	d) Lalit
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3. There are eight friends A, B, C, D, E, F, G, H sitting around the circle facing outward from the centre. G is third to the right of A and is not the neighbor of F and H. D and C are immediate neighbor of G. F is second to the right of B and second to the left of the G. D is not the neighbor of E or F. E is the immediate neighbour of B and H.
- Who among the following are immediate neighbours of F?
 a) B,C b) E,C c) C,A d) H,A
 - Who among the following is sitting third to the right of E?
 a) F b) G c) D d) A
 - Which of the following statements is false?
 a) D is exactly between H and C b) E is opposite to F
 c) C is second to the left of A d) All are false
 - Who among the following is sitting third to the left of A?
 a) E b) H c) D d) G
 - Who among the following is opposite to D?
 a) A b) E c) B d) H
4. A, B, C, D, E, F and G are sitting in a row facing North. F is to the immediate right of E. E is 4th to the right of G. C is the neighbour of B and D. Person who is third to the left of D is at one end.
- Who are to the left of C?
 a) Only B b) G, B & D c) G & B d) D, E, F & A
 - Which of the following statement is not true?
 a) E is to the immediate left of D b) A is at one of the ends
 c) G is to the immediate left of B d) F is second to the right of D
 - Who are the neighbours of B?
 a) C & D b) C & G c) G & F d) C & E
 - What is the position of A?
 a) Between E & D b) Extreme left c) Center d) Extreme right

5. Eleven friends M, N, O, P, Q, R, S, T, U, V and W are sitting in the first row of the stadium watching a cricket match.

T is to the immediate left of P and third to the right of U.

V is the immediate neighbour of M and N and third to the left of S.

M is the second to the right of Q, who is at one of the ends.

R is sitting next to the right of P and P is second to the right of O.

- i. Who is sitting in the center of the row?
 - a) N
 - b) O
 - c) S
 - d) U
- ii. Which of the following people are sitting to the right of S?
 - a) OTPQ
 - b) OTPR
 - c) UNVM
 - d) UOTPR
- iii. Which of the following statements is true with respect to the above arrangement?
 - a) There are three persons sitting between P and S
 - b) W is between M and V.
 - c) N is sitting between V and U
 - d) S and O are neighbours sitting to the immediate right of T
- iv. Who are the immediate neighbours of T?
 - a) O, P
 - b) O, R
 - c) N, U
 - d) V, U
- v. If Q and P, O and N, M and T, and W and R interchange their positions then which of the following pairs of friends is sitting at the ends?
 - a) P and Q
 - b) Q and R
 - c) P and W
 - d) W and R

6. Five girls are sitting on a bench to be photographed. Seema is to the left of Rani and to the right of Bindu. Mary is to the right of Rani. Reeta is between Rani and Mary.

- i. Who is sitting immediate right to Reeta?
 - a) Bindu
 - b) Rani
 - c) Mary
 - d) Seema
- ii. Who is in the middle of the photograph?
 - a) Mary
 - b) Rani
 - c) Reeta
 - d) Seema
- iii. Who is second from the right?
 - a) Mary
 - b) Rani
 - c) Reeta
 - d) Bindu
- iv. Who is second from the left in photograph?
 - a) Reeta
 - b) Mary
 - c) Bindu
 - d) Seema

7. A, B, C, D and E are five men sitting in a line facing south - while M, N, O, P and Q are five ladies sitting in the second line parallel to the first line and are facing North.
- B, who is just next to the left of D, is opposite to Q.
 C and N are diagonally opposite to each other.
 E is opposite to O who is just next right of M.
 P, who is just to the left of Q, is opposite to D. M is at one end of the line.
- Who is sitting third to the right of O?
 a) Q b) N c) M d) Data inadequate
 - If B shifts to the place of E, E shifts to the place of Q, and Q shifts to the place of B, then who will be the second to the left of the person opposite to O?
 a) Q b) P c) E d) D
 - If O and P, A and E & B and Q interchange their positions, then who will be the second person to the right of the person who is opposite to the person second of the right of P?
 a) D b) A c) E d) O
8. Eight students A, B, C, D, E, F, G and H are sitting around a square table facing the centre in such a way that four of them sit at four corners of the square while four sit in the middle of each of the four sides.
- A sits second to the right of F.
 F sits in the middle of one of the sides of the table.
 G who does not sit at any of the corners of the table sits second to the right of D.
 Only two people sit between D and B (taken from one side).
 C is not an immediate neighbor of G.
 H sits second to the left of B.
 E is not an immediate neighbor of G or F.
- Who sits exactly between F and A?
 a) B b) C c) E d) H
 - How many persons sit between A and H, if counted in anti-clockwise direction from A?
 a) 1 b) 2 c) 3 d) 4
 - Four of the following alike a certain way based on their seating positions in the above arrangement and so form a group. Which is the one that does not belong to the group?
 a) E b) F c) A d) D
 - What is the position of F with respect to C?
 a) Third to the left b) Immediate to the right
 c) Second to the left d) Immediate to the left
 - What will come next in the following sequence based upon given seating arrangement?
 AE, EH, HF, _____
 a) FE b) HC c) FD d) DB

9. Eight students A, B, C, D, E, F, G and H are sitting around a square table in such a way four of them sit at four corners while four sit in the middle of each of the four sides. The one who sits at the 4 corners faces the centre and others facing outside.

A who faces the centre sits third to the left of F.

E who faces the centre is not an immediate neighbour of F.

Only 1 person sits between F and G. D sits second to the right of B.

B faces the centre.

C is not an immediate neighbour of A.

- i. Which one does not belong to that group out of five?

a) B b) C c) D d) E

- ii. Which will come in the place of x?

BCE EHA AGD x

a) DFB b) DGA c) DCG d) DBF

- iii. What is the position of G with respect to C?

a) Third to the right b) Fourth to the left
c) Fourth to the right d) Both b & c

- iv. Who sits third to the left of B?

a) H b) A c) G d) F

- v. Which is true from the given arrangement?

a) G faces the centre b) B faces outside c) H faces inside
d) A faces the centre

10. In a class there are seven students (including boys and girls) A, B, C, D, E, F and G. They sit on three benches I, II and III such that at least two students on each bench and at least one girl on each bench.

C, who is a girl student, does not sit with A, E and D.

F the boy student sits with only B.

A sits on the bench I with his best friends.

G sits on the bench III.

E is the brother of C.

- i. How many girls are there out of these 7 students?

a) 3 b) 3 or 4 c) 4 d) Data inadequate

- ii. Which of the following is the group of girls?

a) BAC b) BFC c) BCD d) CDF

- iii. Who sits with C?

a) B b) D c) G d) E

- iv. On which bench there are three students?

a) Bench I b) Bench II c) Bench III d) Bench I or II

11. Eight persons E, F, G, H, I, J, K and L are seated around a square table - two on each side.
 There are 3 ladies who are not seated next to each other.
 J is between L and F. G is between I and F.
 H, a lady member is second to the left of J.
 F, a male member is seated diagonally opposite to K.
 E is a lady member.
 There is a lady member between F and I.
- Who among the following is to the immediate left of F?
 a) G b) I c) J d) H
 - What is true about J and K?
 a) J is male, K is female b) J is female, K is male
 c) Both are female d) Both are male
 - How many persons are seated between K and F?
 a) 1 b) 2 c) 3 d) 4
 - Who among the following are three lady members?
 a) E, H & J b) E, F & G c) E, H & G d) C, H & J
 - Who among the following is seated between E and H?
 a) F b) I c) K d) Cant be determined
12. Eight friends P, Q, R, S, T, U, V and W live on separate floor. Ground floor is no 1, next floor is no 2 and so on.
 V lives on floor no 1. Only 2 people live between V and Q.
 P lives immediately above U. P lives on an even no floor.
 Only one person lives between U and T. U lives above T.
 W lives on an odd no. floor above P.
 S does not live on the topmost floor.
- Who lives on the floor no 2?
 a) W b) S c) Q d) P
 - Who among the following lives below P and what is their floor no?
 a) R – 7 b) Q – 3 c) P – 5 d) U – 5
 - How many people live between R and T?
 a) 1 b) 2 c) 3 d) 4
 - Who lives between R and P?
 a) W b) Q c) V d) U
 - Which is true from the given arrangement?
 a) 8 – W b) 7 – R c) 6 – P d) 5 – S

5. SYLLOGISM

A syllogism is a kind of logical argument that applies deductive reasoning to arrive at a conclusion based on the given statement that are asserted or assumed to be true.

In this topic, questions may contain two or more statements and these statements are followed by two or more conclusions. We have to find out which of the conclusions logically follow from the given statements. The statements have to be taken true even if they seem to be at variance from the commonly known facts. We need to conclude only as per the statement, strictly not beyond that.

For example,

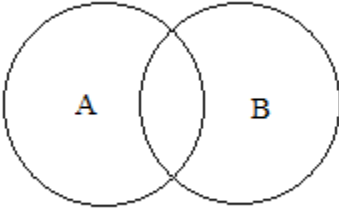
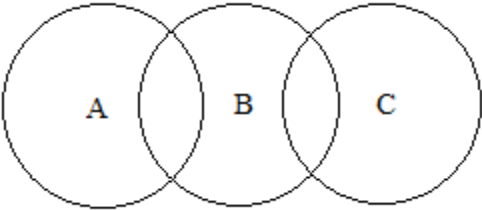
If the given statements are,

- ✓ All birds are beautiful.
- ✓ Pigeon is a bird.

From the given statements, we can know that all birds are beautiful (major premise) and that Pigeon is a bird (minor premise), we may validly conclude that the pigeon is beautiful.

We may draw all the possible Venn Diagrams on the basis of the given statements and then derive the solution from these diagrams.

POSITIVE STATEMENT

<p>TYPE I</p> <p>STATEMENT:</p> <p style="text-align: center;">Some A's are B</p> <div style="text-align: center;">  </div> <p>CONCLUSION:</p> <p style="text-align: center;">Some A's are B</p> <p style="text-align: center;">Some B's are A</p>	<p>TYPE II</p> <p>STATEMENT:</p> <p style="text-align: center;">Some A's are B</p> <p style="text-align: center;">Some B's are C</p> <div style="text-align: center;">  </div> <p>CONCLUSION:</p> <p style="text-align: center;">Some A's are B</p> <p style="text-align: center;">Some B's are A</p> <p style="text-align: center;">Some B's are C</p> <p style="text-align: center;">Some C's are B</p>
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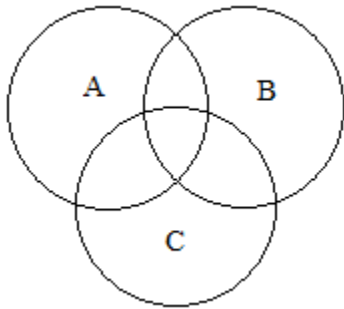
TYPE III

STATEMENT:

Some A's are B

Some B's are C

Some C's are A



CONCLUSION:

Some A's are B

Some B's are A

Some B's are C

Some C's are B

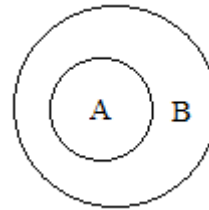
Some A's are C

Some C's are A

TYPE IV

STATEMENT:

All A's are B



CONCLUSION:

All A's are B

Some A's are B

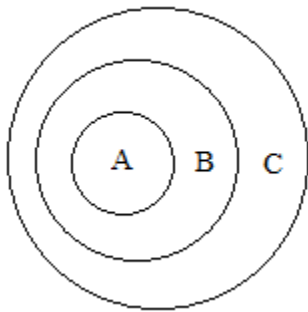
Some B's are A

TYPE V

Statement:

All A's are B

All B's are C



Conclusion:

All A's are B

All A's are C

All B's are C

Some A's are B

Some B's are A

Some B's are C

Some C's are B

Some A's are C

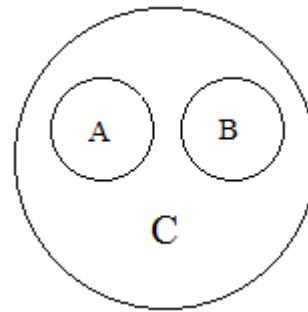
Some C's are A

TYPE VI

Statement:

All A's are C

All B's are C



Conclusion:

All A's are C

All B's are C

Some A's are C

Some B's are C

Some C's are A

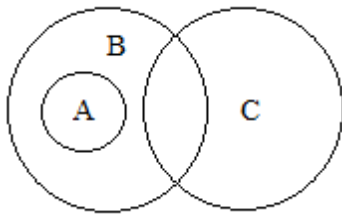
Some C's are B

TYPE VII

Statement:

All A's are B

Some B's are C



Conclusion:

All A's are B

Some A's are B

Some B's are A

Some C's are B

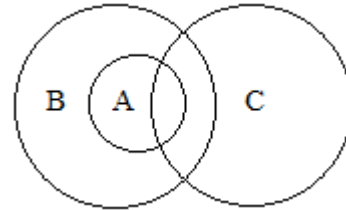
Some B's are C

TYPE VIII

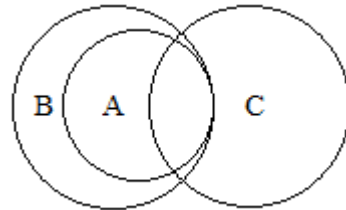
Statement:

All A's are B

Some A's are C



Or



Conclusion:

All A's are B

Some A's are B

Some B's are A

Some B's are C

Some C's are B

Some A's are C

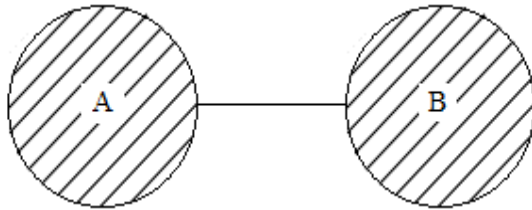
Some C's are A

NEGATIVE STATEMENT

TYPE I

Statement:

All A's are not B



Conclusion:

All A's are not B

All B's are not A

Some A's are not B

Some B's are not A

No A is B

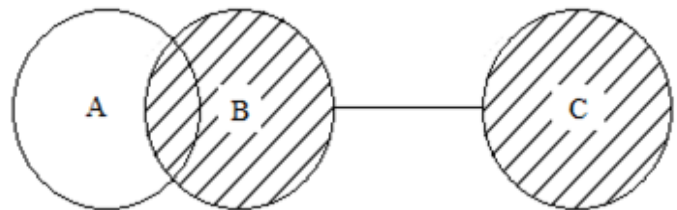
No B is A

TYPE II

Statement:

Some A's are B

All B's are not C



Conclusion:

Some A's are B

Some B's are A

Some B's are not C

Some C's are not B

All B's are not C (or) No B is C

All C's are not B (or) No C is B

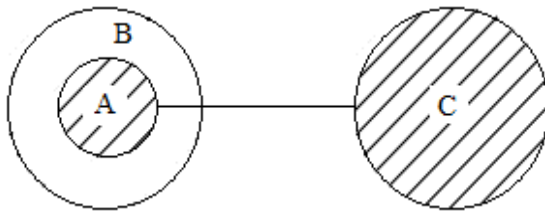
Some A's are not C ***

TYPE III

Statement:

All A's are B

No A is C



Conclusion:

All A's are B

Some A's are B

Some B's are A

All A's are not C

All C's are not A

Some A's are not C

Some C's are not A

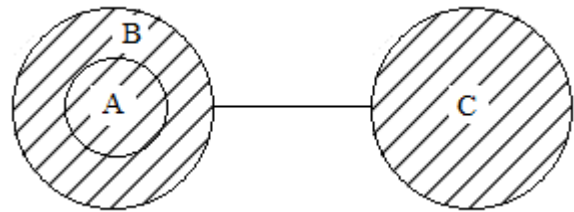
Some B's are not C

TYPE IV

Statement:

All A's are B

No B is C



Conclusion:

All A's are B

Some A's are B

Some B's are A

All B's are not C

All C's are not B

Some B's are not C

Some C's are not B

Some C's are not A

Some A's are not C

All A's are not C

All C's are not A

DRILL

Tick the correct conclusions for the following statements

TYPE I**Statement:**

Some Books are Pens

Conclusion:

- a) Some Pens are Books
- b) All Books are Pens
- c) All Pens are Books
- d) Some Books are not Pens
- e) No Book is Pen
- f) No Pen is Book

TYPE II**Statement:**

Some Books are Pens
Some pens are Erasers

Conclusion:

- a) Some Pens are Books
- b) Some Erasers are pens
- c) Some Erasers are Books
- d) No Eraser is Book
Note: Either (c) or (d)
- e) Some Books are Erasers
- f) No Book is Eraser
Note: Either (e) or (f)
- g) All Pens are Books
- h) All Erasers are pens
- i) All Erasers are Books

TYPE III**Statement:**

Some Engineers are Fathers
Some Fathers are Sons
Some sons are Engineers

Conclusion:

- a) Some Fathers are Engineers
- b) Some sons are Fathers
- c) Some Engineers are sons
- d) All Engineers are Fathers
- e) All Fathers are Sons
- f) All sons are Engineers
- g) All Engineers are sons

TYPE IV**Statement:**

All Dogs are Animals

Conclusion:

- a) Some Dogs are Animals
- b) Some Animals are Dogs
- c) All Animals are Dogs
- d) All Animals are not Dogs
- e) No Dog is Animal

TYPE V**Statement:**

All Men are Boys
All Boys are Students

Conclusion:

- a) Some Men are Boys
- b) Some Boys are Men
- c) Some Boys are Students
- d) Some Students are Boys
- e) All Men are Students
- f) Some Men are Students
- g) Some Students are Men
- h) All Students are Men
- i) No Student is Men

TYPE VI**Statement:**

All Chairs are Furniture
All Tables are Furniture

Conclusion:

- a) Some Furniture are Chairs
- b) Some Tables are Furniture
- c) All Furniture are Chairs
- d) All Furniture are Tables
- e) No Chair is Table
- f) No Table is Chair
- g) Some Furniture are not Chairs
- h) Some Furniture are not Tables

TYPE VII**Statement:**

All Apples are Fruits
Some Fruits are Vegetables

Conclusion:

- Some Apples are Fruits
- All Fruits are Apples
- Some Vegetables are Fruits
- Some Apples are Vegetables
- Some Vegetables are Apples
- All Fruits are Vegetables

TYPE VIII**Statement:**

All Bikes are Vehicles
Some Bikes are Pulsar

Conclusion:

- Some Vehicles are Pulsars
- Some Pulsars are Bikes
- All Vehicles are Bikes
- All Pulsars are Vehicles
- All Bikes are Pulsars
- No Bikes are Pulsars
- All Bikes are Vehicles
- Some Vehicles are not Pulsars
- Some Pulsars are not Bikes

NEGATIVE STATEMENT**TYPE I****Statement:**

All Cakes are not Cookies

Conclusion:

- No Cakes are Cookies
- All Cookies are not Cakes
- Some Cakes are Cookies
- Some Cakes are not Cookies
- Some Cookies are Cakes
- All Cookies are Cakes
- Some Cookies are not Cakes

TYPE III**Statement:**

All Bangles are Ring
No Bangle is Coin

Conclusion:

- All Bangles are not Coins
- All Rings are not Coins
- Some Rings are not Coins
- Some Rings are Coins
- Some Rings are Bangles
- Some Bangles are Rings
- Some Coins are not Bangles
- No Coin is Bangle
- Some Bangles are not Coins
- All Rings are Bangles

TYPE II**Statement:**

Some Toys are Boxes
All Boxes are not Metals

Conclusion:

- Some Boxes are Toys
- No Metal is Box
- No Metal is Toy
- No Toy is Box
- No Toy is Metal
- All Toys are not Metals
- Some Toys are not Metals
- Some Toys are Metals

TYPE IV**Statement:**

All Pastries are cakes
No Cake is Ice-cream

Conclusion:

- Some Pastries are cakes
- Some cakes are Pastries
- All Pastries are not cakes
- Some Ice-creams are cakes
- Some Pastries are Ice-creams
- Some cakes are Ice-creams
- Some cakes are not Ice-creams
- Some Pastries are not Ice-creams
- Some cakes are not Pastries

PRACTICE QUESTIONS

Directions to Solve (1-20)

In each of the following questions two statements are given and these statements are followed by two conclusions numbered I and II. You have to consider the given two statements to be true, even if they seem to be at variance from commonly known facts. Read the conclusions and then decide which of the given conclusions logically follows from the two given statements, disregarding commonly known facts.

Give answer:

- (A) If only (I) conclusion follows
- (B) If only (II) conclusion follows
- (C) If either (I) or (II) follows
- (D) If neither (I) nor (II) follows and
- (E) If both (I) and (II) follow.

1. **Statements:**

Some actors are singers.
All the singers are dancers.

Conclusions:

- I. Some actors are dancers.
- II. No singer is actor.

2. **Statements:**

All harmoniums are instruments.
All the instruments are flutes.

Conclusions:

- I. All the flutes are instruments.
- II. All the harmoniums are flutes.

3. **Statements:**

Some mangoes are yellow.
Some fruits are mangoes.

Conclusions:

- I. Some mangoes are fruits.
- II. Fruit is a yellow.

4. **Statements:**

Some ants are parrots.
All the parrots are apples.

Conclusions:

- I. All the apples are parrots.
- II. Some ants are apples.

5. **Statements:**

Some papers are pens.
All the pencils are pens.

Conclusions:

- I. Some pens are pencils.
- II. Some pens are papers.

6. **Statements:**

All flowers are candles.
All lanterns are candles.

Conclusions:

- I. Some candles are lanterns.
- II. Some flowers are lanterns.

7. **Statements:**

All sides are lengths.
No length is breadth.

Conclusions:

- I. All lengths are sides
- II. No breadth is side

8. **Statements:**

Some men are bachelors.
All bachelors are blessed.

Conclusions:

- I. Some men are blessed.
- II. At least some blessed are bachelors.

9. **Statements:**

Some pictures are beds.
All beds are trees.

Conclusions:

- I. Some pictures are trees.
- II. At least some trees are beds.

10. **Statements:**

Some ninjas are fighters.
No fighter is liar.

Conclusions:

- I. No ninja is liar.
- II. At least some ninjas are liars.

11. Statements:

Some mangos are brinjals.
Some carrots are brinjals.

Conclusions:

- I. All mangos are carrots.
- II. At least some brinjals are not carrots.

12. Statements:

No cake is ginger.
Some gingers are garlic.

Conclusions:

- I. No cake is garlic.
- II. Some garlicks are not cakes.

13. Statements:

No pizza is burger.
No nuggets is burger.

Conclusions:

- I. Some pizzas are not nuggets.
- II. Some burgers are nuggets.

14. Statements:

All fingers are levers.
Some levers are rods.

Conclusions:

- I. Some rods are levers.
- II. No finger is rod.

15. Statements:

No book is pencil.
All pencils are pens.
No pen is blue.

Conclusions:

- I. No blue is pencil.
- II. All books are pen

16. Statements:

All eggs are tomatoes.
No tomato is potato.
All potatoes are goods.

Conclusions:

- I. Some tomatoes may be goods.
- II. All eggs being goods is a possibility.

17. Statements:

All princes are kings.
All braves are princes.
No queen is king.

Conclusions:

- I. All princes being brave is a possibility.
- II. Atleast some kings are brave.

18. Statements:

All princes are kings.
All braves are princes.
No queen is king.

Conclusions:

- I. A prince can never be a queen.
- II. All those princes who are kings are queens.

19. Statements:

All squares are circles.
All circles are triangles.
Some rectangles are triangles.

Conclusions:

- I. All circles being rectangles is a possibility.
- II. Some triangles are squares.

20. Statements:

All squares are circles.
All circles are triangles.
Some rectangles are triangles

Conclusions:

- I. All squares being rectangle is a possibility.
- II. At least some rectangles are circles.

Directions to Solve (21-25)

In each of the following questions two statements are given and they are followed by four conclusions (I), (II), (III) and (IV). Choose the conclusions, using the given options, which logically follow from the given statements.

21. Statements:

- No door is dog.
- All dogs are cats.

Conclusions:

- III. Some doors are cats.
- IV. No cat is door.
- V. Some cats are dogs.
- VI. All cats are dogs.

- a) Only (I) and (III)
- b) Only (II) and (IV)
- c) Either (I) or (II) and (III)
- d) Only (III)
- e) All the four

22. Statements:

- All green are blue.
- All blue are white.

Conclusions:

- I. Some blue are green.
- II. Some white are green.
- III. Some green are not white.
- IV. All white are blue.

- a) Only (I) and (II)
- b) Only (I) and (III)
- c) Only (I) and (IV)
- d) Only (II) and (IV)
- e) None of the above

23. Statements:

- All men are vertebrates.
- Some mammals are vertebrates.

Conclusions:

- I. All men are mammals.
- II. All mammals are men.
- III. Some vertebrates are mammals.
- IV. All vertebrates are men.

- a) Only (IV)
- b) Only (II)
- c) Only (III)
- d) Only (I)
- e) Only (I) and (III)

24. Statements:

- All the phones are scales.
- All the scales are calculators.

Conclusions:

- I. All the calculators are scales.
- II. All the phones are calculators
- III. All the scales are phones.
- IV. Some calculators are phones.

- a) Only (I) and (IV)
- b) Only (III) and (IV)
- c) Only (II) and (IV)
- d) Only (I) and (II)
- e) Only (I) and (III)

25. Statements:

- Some tables are T.V
- Some T.V are radios.

Conclusions:

- I. Some tables are radios.
- II. Some radios are tables.
- III. All the radios are T.V.
- IV. All the T.V are tables.

- a) Only (2) and (4)
- b) Only (1) and (3)
- c) Only (4)
- d) Only (1) and (4)
- e) None of the four.

6. SEQUENCE & SERIES

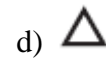
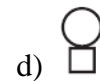
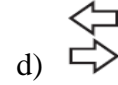
DRILL

1. 1, 3, 6, 10, 15, _____
2. 6, 12, 36, 144, _____, 4320
3. 10, 5, 13, 10, 16, 20, 19, _____, _____
4. 1, 4, 9, 16, 25, 36, _____
5. 2, 4, 12, 48, 240, _____
6. 11, 13, 17, 19, 23, 29, 31, 37, 41, _____
7. 16, 33, 65, 131, 261, _____
8. 5, 12, 23, 50, 141, _____
9. 1, 1, 4, 8, 9, 27, 16, 64, _____, _____
10. 11, 6, 5, 9, 16, _____
11. A, Z, C, X, E, _____, _____
12. SCD, TEF, UGH, _____, WKL
13. B2CD, _____, BCD4, B5CD, BC6D
14. ADC, EHG, ILK, MPO, _____
15. A, D, H, M, S, _____

PRACTISE QUESTIONS

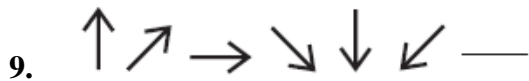
1. 8, 24, 12, 36, 18, 54, _____
2. 61, 133, 277, 565, _____
3. 17, 18, 40, 129, 532, _____
4. 16, 745, 1257, 1600, 1816, 1941, _____
5. 9, 90, 154, 203, 239, 264, _____
6. 12, 20, 30, 42, _____, 72
7. 4, 5, 9, 18, 34, 59, 95, _____
8. 10, 17, 26, 37, _____, 65
9. 135, 15, 120, 17.1, 102.6, _____
10. 9, 5, 6, 10.5, _____, 60, 183

7. SYMBOLS & SERIES





- a) b) c) d)



- a) b) c) d)



- a) b) c) d)



- a) b) c) d)



- a) b) c) d)



- a) b) c) d)



- a) b) c) d)

8. CRYPT ARITHMETIC

Important properties to Solve Crypt Arithmetic Problems

- ✓ Each alphabet takes only one number from 0 to 9.
- ✓ Starting character of number can't be zero. For example, 0341 should be simply 341.
- ✓ The problem will have only one solution.
- ✓ Addition of two even or two odd numbers is always even.
- ✓ Sum of two same numbers must be an even number.
- ✓ Carry over can be
 - Maximum 1 when we will add two numbers
 - Maximum 2 when we will add three numbers
 - Maximum 3 when we will add four numbers
- ✓ If sum of A & B is A, then the value of B must be 0 (zero) if A and B are unit digits.
- ✓ If sum of A & B is A, then the value of B may be 0 or 9 if A and B are other than unit digits.
- ✓ Addition of two numbers with N digits, if the result is in N+1 digits then the left most digit must be 1.
- ✓ Once all the characters/letters are replaced with numbers, their arithmetic operations must be correct.

Example 1:

If "EAT + THAT = APPLE", what is the sum of A + P + P + L + E ?

a) 13

b) 14

c) 12

d) 15

Solution:

$$\begin{array}{r}
 \text{E A T} \\
 \text{T H A T} \\
 \hline
 \text{A P P L E} \\
 \hline
 \end{array}$$

From the given data, the value of A will be 1 because it is the only carry-over possible from the sum of 2 single digit number.

The maximum value that T can take is 9 and there should be a carryover for T to give sum as A P (2 digit number).

So $T = 9$, $P = 0$, $A = 1$. $T + T = 18$, the value of E is 8 and 1 will be a carry over to the next column.

Then, $1 + A + A = L = 3$.

And finally, $E + H = P$, i.e., $8 + H = 10 \Rightarrow H = 2$

Hence, $819 + 9219 = 10038$

$\Rightarrow A + P + P + L + E = 1 + 0 + 0 + 3 + 8 = 12$.

Example 2:

“HERE = COMES – SHE”, assume $S = 8$. Find values of $R + H + O$.

- a) 15 b) 18 c) 14 d) 12

Solution:

$$\begin{array}{r} \text{H E R E} \\ \text{S H E} \\ \hline \text{C O M E S} \\ \hline \end{array}$$

Given that, $S = 8$

i.e., $E + E = 8$

So, $E = 4$

Then, $H + 1 = 10$

So, $H = 9, O = 0, C = 1,$

Now, $R + H = E$ i.e., $R + 9 = 14 \Rightarrow R = 5$

So, $R + H = 5 + 9 = 14$ which leaves carry 1 and it will be a carry over to the next column.

Then, $1 + E + S = M$

i.e., $1 + 4 + 8 = 13$

So, $M = 3$ and 1 will be a carry over to the next column.

Hence, $R + H + O = 5 + 9 + 0 = 14$

DRILL

- TO + GO = OUT. Find the value of $T + G + O + U$.
a) 15 b) 11 c) 7 d) 18
- THIS + IS = HERE. Find all the values.
a) 13 b) 14 c) 12 d) 15
- NO + GUN + NO = HUNT, find the value of HUNT.
a) 1082 b) 1802 c) 1208 d) 1280

PRACTICE QUESTIONS

- IF EAT + THAT = APPLE, What is $A + P + P + L + E$?
a) 13 b) 14 c) 12 d) 15
- If POINT + ZERO = ENERGY, then $E + N + E + R + G + Y = ?$
a) 17 b) 13 c) 19 d) 18
- YOUR + YOU = HEART (Given $O = 4$) Value of $Y + U + R + E$ is ?
a) 15 b) 16 c) 17 d) 18
- USSR + USA = PEACE, what is the value of $P + E + A + C + E$?
a) 9 b) 10 c) 11 d) 12
- MAC + MAAR = JOCKO, find value of $3A + 2M + 2C$?
a) 31 b) 36 c) 33 d) 38
- If TOM + NAG = GOAT, then find the value of $G + O + A + T$.
a) 15 b) 12 c) 14 d) Cannot be determined

9. DATA SUFFICIENCY

Data sufficiency questions consist of a question followed by two statements. The test-taker is assessed based on the ability to decide whether the information in the statements (taken singly or together) is sufficient to answer the question.

Data sufficiency tricks

Step 1: Examine the Question.

Step 2: Consider each statement individually.

Step 3: If necessary combine the two statements.

Directions to Solve: In each of the questions below consists of a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statements are sufficient to answer the question. Read both the statements and give the answer as

- (A) If the data in statement I alone are sufficient to answer the question, while the data in statement II alone are not sufficient to answer the question.
- (B) If the data in statement II alone are sufficient to answer the question, while the data in statement I alone are not sufficient to answer the question.
- (C) If the data either in statement I alone or in statement II alone are sufficient to answer the question.
- (D) If the data given in both statements I and II together are not sufficient to answer the question.
- (E) If the data in both statements I and II together are necessary to answer the question.

Example 1:

In which year was Rahul born?

Statements:

- I. Rahul at present is 25 years younger to his mother.
- II. Rahul's brother, who was born in 1964, is 35 years younger to his mother.

- A) I alone is sufficient while II alone is not sufficient
- B) II alone is sufficient while I alone is not sufficient
- C) Either I or II is sufficient
- D) Neither I nor II is sufficient
- E) Both I and II are sufficient

Answer: Option E

Explanation:

From both I and II, we find that Rahul is $(35 - 25) = 10$ years older than his brother, who was born in 1964.

So, Rahul was born in 1954.

Example 2:

What will be the total weight of 10 poles, each of the same weight?

Statements:

- I. One-fourth of the weight of each pole is 5 kg.
- II. The total weight of three poles is 20 kilograms more than the total weight of two poles.

- A) I alone is sufficient while II alone is not sufficient
- B) II alone is sufficient while I alone is not sufficient
- C) Either I or II is sufficient
- D) Neither I nor II is sufficient
- E) Both I and II are sufficient

Answer: Option C

Explanation:

From I, we conclude that,

$$\text{Weight of each pole} = (4 \times 5) \text{ kg} = 20 \text{ kg.}$$

$$\text{So, total weight of 10 poles} = (20 \times 10) \text{ kg} = 200 \text{ kg.}$$

From II, we conclude that,

$$\text{Weight of each pole} = (\text{weight of 3 poles}) - (\text{weight of 2 poles}) = 20 \text{ kg.}$$

$$\text{So, total weight of 10 poles} = (20 \times 10) \text{ kg} = 200 \text{ kg.}$$

Example 3:

How is Tanya related to the man in the photograph?

Statements:

- I. Man in the photograph is the only son of Tanya's grandfather.
- II. The man in the photograph has no brothers or sisters and his father is Tanya's grandfather.

- A) I alone is sufficient while II alone is not sufficient
- B) II alone is sufficient while I alone is not sufficient
- C) Either I or II is sufficient
- D) Neither I nor II is sufficient
- E) Both I and II are sufficient

Answer: Option B

Explanation:

From I, we conclude that the man is the only son of Tanya's grandfather i.e. he is Tanya's father or Tanya is the man's daughter.

From II, we conclude that the man's father is Tanya's grandfather. Since the man has no brothers or sisters, so he is Tanya's father or Tanya is the man's daughter.

Example 4:

How many children does M have?

Statements:

- I. H is the only daughter of X who is wife of M.
- II. K and J are brothers of M.

- A) I alone is sufficient while II alone is not sufficient
- B) II alone is sufficient while I alone is not sufficient
- C) Either I or II is sufficient
- D) Neither I nor II is sufficient
- E) Both I and II are sufficient

Answer: Option D

Explanation:

From I, we conclude that H is the only daughter of M. But this does not indicate that M has no son. The information given in II is immaterial.

Example 5:

What is Suman's rank from the top in a class of forty students?

Statements:

- I. Suman is 3 ranks below Deepak from the top.
- II. Deepak's rank from the bottom is 23.
- III. Suman is 3 ranks above Deepak from the bottom.

- A) Any two of the three
- B) Only I and II
- C) Only II and III
- D) All I, II and III
- E) Only II and either I or III

Answer: Option E

Explanation:

From II, we conclude that in a class of 40, Deepak ranks 23rd from the bottom i.e. 18th from the top.

From I and II, we find that Suman is 3 ranks below 18th rank from the top i.e. she ranks 21st from the top.

From II and III, we find that Suman is 3 ranks above 23rd rank from the bottom i.e. she ranks 20th from the bottom or 21st from the top.

DRILL

1. How much was the total sale of the company?

Statements:

- I. The company sold 8000 units of product A each costing Rs. 25.
 II. This company has no other product line.

- A) I alone is sufficient while II alone is not sufficient
 B) II alone is sufficient while I alone is not sufficient
 C) Either I or II is sufficient
 D) Neither I nor II is sufficient
 E) Both I and II are sufficient

2. The last Sunday of March, 2006 fell on which date?

Statements:

- I. The first Sunday of that month fell on 5th.
 II. The last day of that month was Friday.

- A) I alone is sufficient while II alone is not sufficient
 B) II alone is sufficient while I alone is not sufficient
 C) Either I or II is sufficient
 D) Neither I nor II is sufficient
 E) Both I and II are sufficient

3. How is 'No' coded in the code language?

Statements:

- I. "Ne Pa Sic Lo" means "But No None And" and
 "Pa Lo Le Ne" means "If None And But".
 II. "Le Se Ne Sic" means "If No None Will" and
 "Le Pi Se Be" means "Not None If All".

- A) I alone is sufficient while II alone is not sufficient
 B) II alone is sufficient while I alone is not sufficient
 C) Either I or II is sufficient
 D) Neither I nor II is sufficient
 E) Both I and II are sufficient

4. Who among P, Q, T, V and M is exactly in the middle when they are arranged in ascending order of their heights?

Statements:

- I. V is taller than Q but shorter than M.
 II. T is taller than Q and M but shorter than P.

- A) I alone is sufficient while II alone is not sufficient
 B) II alone is sufficient while I alone is not sufficient
 C) Either I or II is sufficient
 D) Neither I nor II is sufficient
 E) Both I and II are sufficient

10. STATEMENT AND CONCLUSION

Tips and Tricks to solve Statement and Conclusion

- ✓ If there are two or more sentences which are used to frame a statement, then the sentences must be interrelated and mutual contradiction should be there.
- ✓ Do not look for truthful notions. The information provided in the statement is the only requirement for a candidate to answer the question. No assumptions must be made.
- ✓ Read the statement carefully and look for keywords which are common between the statement and the conclusions
- ✓ If there is more than one conclusion which is applicable for the statement, candidates must ensure that the conclusions they opt have some relation with each other.

Examples:

1. Statement:

Panel has decided to shortlist those candidates having minimum first division (60%) throughout their career. Meena has secured 65% in 10th and 59% in 12th. She graduated with 85% percentage.

Conclusions:

- I. Meena will be shortlisted.
- II. Meena will not be shortlisted.

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

Answer: B

Explanation:

Here we have to carefully arrive at the conclusion. Meena has secured more than 60% in both graduation and 10th. However; her 12th percentage is not above 60%. So, she will not be shortlisted. In this we derived the conclusion indirectly but you can see it is not that much difficult.

2. Statement:

Only smart and intelligent people are invited into the conference. Shakthi has been invited to attend the conference.

Conclusions:

- I. Shakthi is smart.
- II. Shakthi is intelligent.

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

Answer: C

3. **Statement:**

In a T20 match played between India and Australia, the total runs made by the Indian team were 200. 160 runs out of 200 runs were made by spinners.

Conclusions:

- I. 80% of the team consists of spinners
- II. The opening batsmen were spinners

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

Answer: D

Explanation: Neither conclusion is logically correct with regard to the statement.

4. **Statement:**

The panel asked 5 members to resign from job or else they will be facing forceful rejection. Four of them have resigned till this evening.

Conclusions:

- I. One manager will resign tomorrow.
- II. The panel will terminate him tomorrow.

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

Answer: E

Explanation: Any of the above two conclusion can happen.

DRILL1. **Statement:**

Ram is the only son of Suresh and Mina.

Conclusions:

- I. Suresh and Mina have one son.
- II. Ram has one sister.

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

2. **Statement:**

The Team Manager humiliated Varun in front of his colleagues on a Monday morning.

Conclusions:

- I. The Team Manager did not like Varun
- II. Varun was not as popular as his other colleagues

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

3. **Statement:**

Bad behaviour of a student can cause defame to both the student and the school in which he is studying.

Conclusions:

- I. Student showing bad behaviour should be removed from the school immediately.
- II. By being strict, we cannot change the behaviour of a student.

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

4. **Statement:**

The distance of 900 km by road between Bombay and Jafra will be reduced to 280 km by sea. This will lead to a saving of Rs. 7.92 crores per annum on fuel.

Conclusions:

- I. Transportation by sea is cheaper than that by road.
- II. Fuel must be saved to the greatest extent

- A) Only Conclusion I is true
- B) Only Conclusion II is true
- C) Both Conclusion I and II are true
- D) Neither Conclusion I nor II is true
- E) Either Conclusion I or II is true

11. STATEMENT AND ARGUMENTS

Argument is a fact that is being given by a person to support or oppose a statement. In this type of questions, a statement concerned with an issue is given, followed by certain arguments in favor or against that statement. You have to identify the logical correct arguments.

Generally, both the arguments are contrary to each other and refer to the positive and negative results of the action as mentioned in the statement issue.

Arguments can be of two types

1. Strong Argument:

An argument is called strong, if it touches the practical and real aspect of the situation as described in the statement. It is backed up by reasons and facts related to the situation.

2. Weak Argument:

An argument is called weak, if it is not directly related to the given statement and it does not address all the points put forward in the given statement. A weak argument is of minor importance or may be related to trivial aspect of the statement.

Important things to Remember

Following points should be taken into consideration while choosing a strong argument.

- ✓ A strong argument should give the realistic diagnosis of the situation described in the statement.
- ✓ An argument based on universal truth is always strong.
- ✓ A strong argument should give deep analysis of the topic dealing with the statement.
- ✓ A strong argument should relate with the statement and be supported up by facts or established notions.

Examples:

Each of the following questions consists of one statement followed by two arguments. You have to decide which one of them is stronger and which one of them is weak.

1. Statement:

Should all the new and big industries be established in Kolkata?

Arguments:

- I. Yes, more job opportunities will be created.
- II. No, the pollution of the city will increase further.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

Answer: C

Explanation:

Here either of the any two can happen. Employment will increase upon the opening of new industries but simultaneously pollution will also increase.

2. **Statement:**

Should strikes and dharnas in the field of hospital be banned?

Arguments:

- I. Yes, it is hampering the hospital and patients are suffering a lot.
- II. No, the employees must get their dues in proper manner.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

Answer: E

Explanation:

Both are strong because strikes are hampering the patients. At the same time the strikes are for salaries and the doctors must be satisfied with their jobs.

3. **Statement:**

Should India support all the international policies of United States of America?

Arguments:

- I. No. Many other powerful countries do not support the same.
- II. Yes. This is the only way to gain access to USA developmental funds.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

Answer: D

Explanation:

Here none of the arguments is strong. Our country cannot support USA's policies blindly without analysis, just to gain monetary help. Also, we should not withdraw our support without considering the policies, just because some other nations have done so.

4. **Statement:**

Should all government work for job must be online?

Arguments:

- I. Yes, this will reduce paperwork and beneficial for environment.
- II. Yes, this process will be transparent and corruption free.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

Answer: E

Explanation: Both the arguments are strong and in context of the statement.

DRILL**1. Statement:**

Will the newly elected CMC mayor fulfill her promises?

Arguments:

- I. Yes, she has to otherwise her existence as mayor will be in danger.
- II. No, candidates elected and after getting power and positions forget their promises.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

2. Statement:

Should school education in poor states like Bihar, Assam, Odisha be made free?

Arguments:

- I. Yes, as these are poor states, this is the one and only way to improve literacy.
- II. No, this might have adverse impact on the economy.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

3. Statement:

Should our government bring down the age of retirement in public services?

Arguments:

- I. Yes, as this may create new vacancies for unemployed and talented ones to serve their country.
- II. No, government officers may have their own family problems which may not be solved if retirement is taken early.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

4. Statement:

Should our government increase the age-limit for retirement in the government services?

Arguments:

- I. Yes, America and all the Scandinavian countries have done it.
- II. Yes, Indian people live long and it is the actual demand of lakh's of employees.

- A) Only argument I is strong.
- B) Only argument II is strong.
- C) Either I or II is strong.
- D) Neither I nor II is strong.
- E) Both I and II are strong.

12. ANALOGY

One literary term that encompasses most types of comparison is known as an Analogy. Analogy implies likeness or parallelism in relations rather than in appearance or qualities. Some common synonyms of analogy are likeness, resemblance, similarity, and similitude.

Verbal Analogies

Verbal Analogies is a type of verbal ability questions. A question has two words with a certain relationship to each other, and it is followed by four pairs of words. You are required to choose one pair in which the words have the same relationship.

- ✓ White : Black :: Up : Down
- ✓ Short : Light :: Long : Heavy
- ✓ Tree : Leaf :: Flower : Petal
- ✓ Hammer : Nail :: Comb : Hair

1. Dog : Puppy :: Cat : ?
2. Bees : Hive :: Bears : ?
3. Speak : Sing :: Walk : ?
4. Chef : Food :: Sculptor : ?
5. Like : Love :: Dislike : ?
6. Pen : Poet :: Needle : ?

Answers:

1. Kitten
2. Den
3. Dance
4. Stone
5. Hate
6. Tailor

Examples

1. Odometer is to mileage as compass is to

a) Speed	b) Hiking	c) Needle	d) Direction
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Answer: d) direction

Explanation:

Odometer is an instrument used to measure mileage. Compass is an instrument used to determine direction. Other options are incorrect because none is an instrument.

2. Cup is to coffee as bowl is to

a) Dish	b) Soup	c) Spoon	d) Food
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Answer: b) Soup

Explanation:

Coffee goes into a cup and soup goes into a bowl. Choices a and c are incorrect because they are other utensils. Choice d is incorrect because food is too general.

3. Reptile is to lizard as flower is to
 a) Petal b) Stem c) Daisy d) Alligator

Answer: *c) Daisy*

Explanation:

A lizard is a type of reptile; a daisy is a type of flower. Choices a and b are incorrect because a petal and a stem are parts of a flower, not types of flowers. Choice d is incorrect because an alligator is another type of reptile, not a type of flower.

4. Play is to actor as concert is to
 a) Symphony b) Musician c) Piano d) Percussion

Answer: *b) Musician*

Explanation:

An actor performs in a play. A musician performs at a concert. Choices a, c, and d are incorrect because none is people who perform.

5. Exercise is to gym as eating is to
 a) Food b) Dieting c) Fitness d) Restaurant

Answer: *d) Restaurant*

Explanation:

A gym is a place where people exercise. A restaurant is a place where people eat. Food is not the answer because it is something people eat, not a place or location where they eat. Choice b and c are incorrect because neither represents a place where people eat.

6. BINDING : BOOK
 a) Artist: Carpenter b) Criminal: gang
 c) Nail: Hammer d) Frame: Picture

Answer: *d) Frame: Picture*

Explanation: A binding surrounds a book; a frame surrounds a picture.

7. EXPLORE : DISCOVER
 a) Read: Skim b) Research: Learn
 c) Write: Print d) Think: Relate

Answer: *b) Research: Learn*

Explanation: One explores to discover; one researches to learn.

8. DEPRESSED : SAD
 a) Towering: Cringing b) Rapid: Plodding
 c) Progressive: Regressive d) Exhausted: Tired

Answer: *d) Exhausted: Tired*

Explanation:

Depressed is an intensification of sad; exhausted is an intensification of tired.

DRILL

1. Dusk: Evening : : Dawn: ?
a) Night b) Light c) Fog d) Morning
2. Paw: Cat then Hoof: ?
a) Dog b) Elephant c) Horse d) Lamb
3. The evening is to Morning as Dinner is to?
a) Night b) Breakfast c) Lunch d) Brunch
4. Hairdresser is to scissors as a butcher is to?
a) Meat b) Spoon c) Knife d) None of the above
5. Gun is to bullet as a bow is to?
a) Arrow b) Wood c) Fly d) Sky
6. After: Before
a) Hot: Cold b) Summer: Winter
c) Sweet: Sour d) Successor: Predecessor
7. Ten: Decimal
a) Two: Binary b) Four: Rectangle
c) Three: Triple d) Two: Double
8. College: Students
a) Hospital: Patients b) Restaurant: Food
c) Game: Player d) Night: Sleep
9. Clock: Time
a) Car: Engine b) Summer: Hot
c) Thermometer: Temperature d) Barometer: Weight
10. Trial: Jury
a) Election: Voting b) Dispute: Arbiter
c) Doctor: Hospital d) Lawyer: Justice
11. Pork: PIG
a) Sea: Fish b) Fruits: Vegetables
c) Mutton: Sheep d) Poultry: Chicken
12. Ducks: Quack
a) Bears: Grunt b) Cat: Roar
c) Dogs: Bark d) Lion: Predator

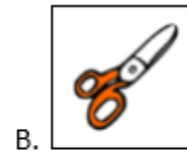
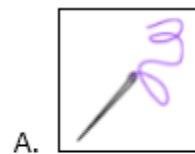
Picture Analogies

This type of reasoning question is similar to word analogies and is solved in the same way as you solve word analogies. The question has a set of two pictures that are related to each other. This set is followed by another set; a picture and an empty box.

The question is followed by four answer choices in the form of pictures. You have to choose one picture that would go in the empty box so that pictures in the second set are related to each other in the same way as the pictures in the first set are related to each other.

Examples

1.

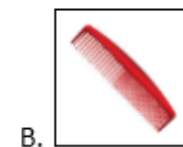
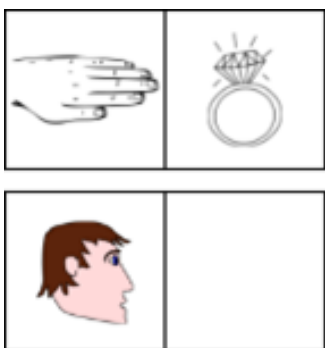


Answer: A

Explanation:

A color palette is to a color brush as a reel of a thread is to a sewing needle, i.e., color brush and sewing needle are the tools needed to perform the task. So, the needle will go in the empty box.

2.

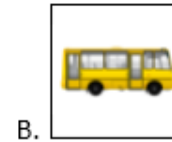


Answer: C

Explanation:

A ring is worn on the hand and a cap is worn on the head, so the cap would go in the empty box.

3.

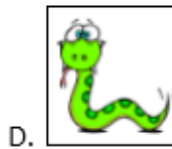
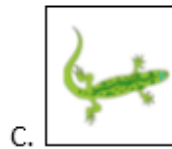
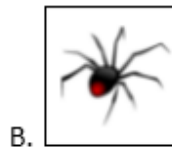


Answer: A

Explanation:

A telephone is to a stamped letter as an airplane is to a bus. A telephone and letter are both forms of communication. An airplane and bus are both forms of transportation.

4.

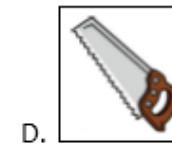
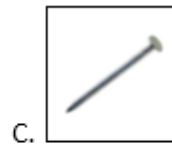


Answer: A

Explanation:

A toddler is to an adult as a caterpillar is to a butterfly. This relationship shows the young and the adult. The caterpillar is an early stage of the adult butterfly.

5.



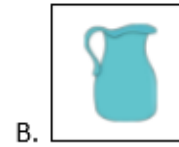
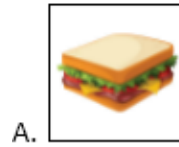
Answer: D

Explanation:

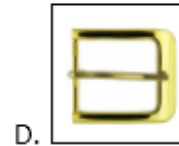
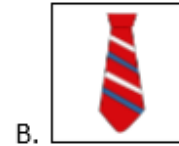
Guitar and harmonium are musical instruments. Similarly, hammer and saw are carpentry tools. So, the saw will enter into the empty box.

DRILL

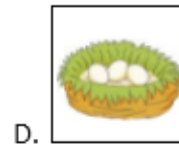
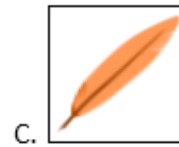
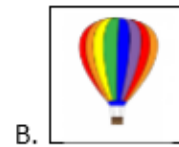
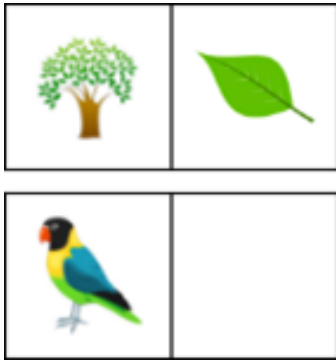
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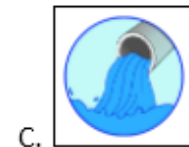
2.



3.



4.



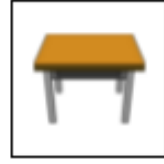
5.



A.



B.



C.



D.

